

Andy Sproles, ORNL

# Microscopic models of nuclear structure: from dripline to dripline

**Wouter Ryssens**, G. Grams, M. Bender and S. Goriely

3th of July 2023



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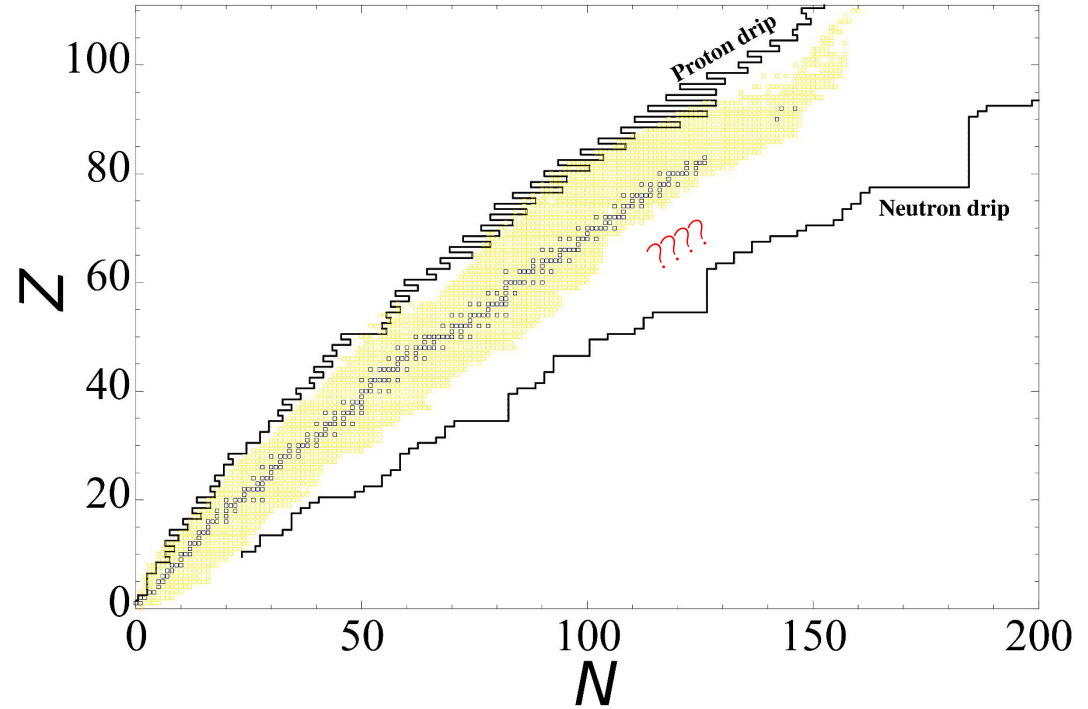


LA LIBERTÉ DE CHERCHER

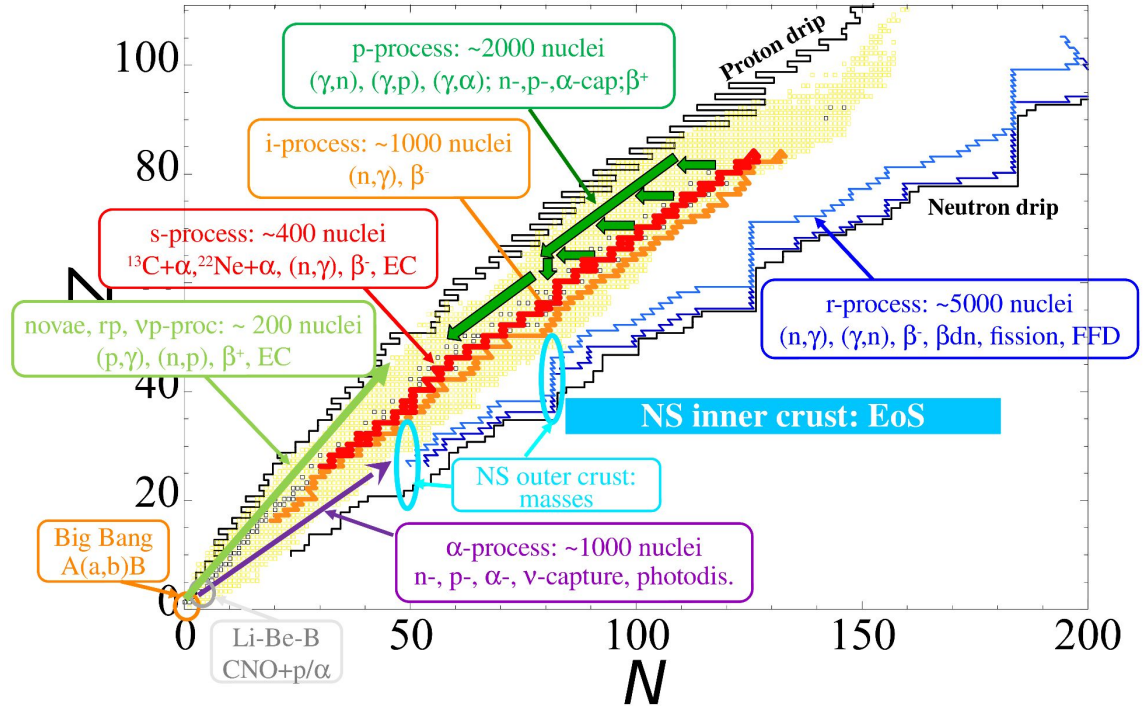


EOS  
THE EXCELLENCE  
OF SCIENCE

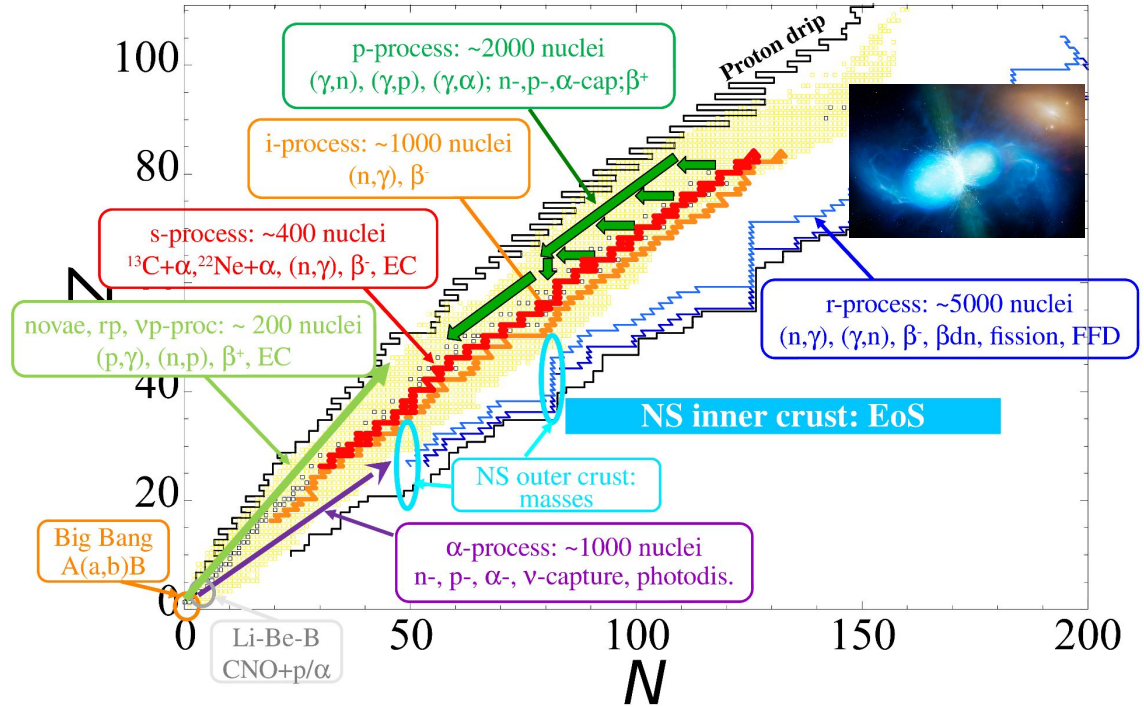
# The nuclear chart...



# The nuclear chart and the processes traversing it



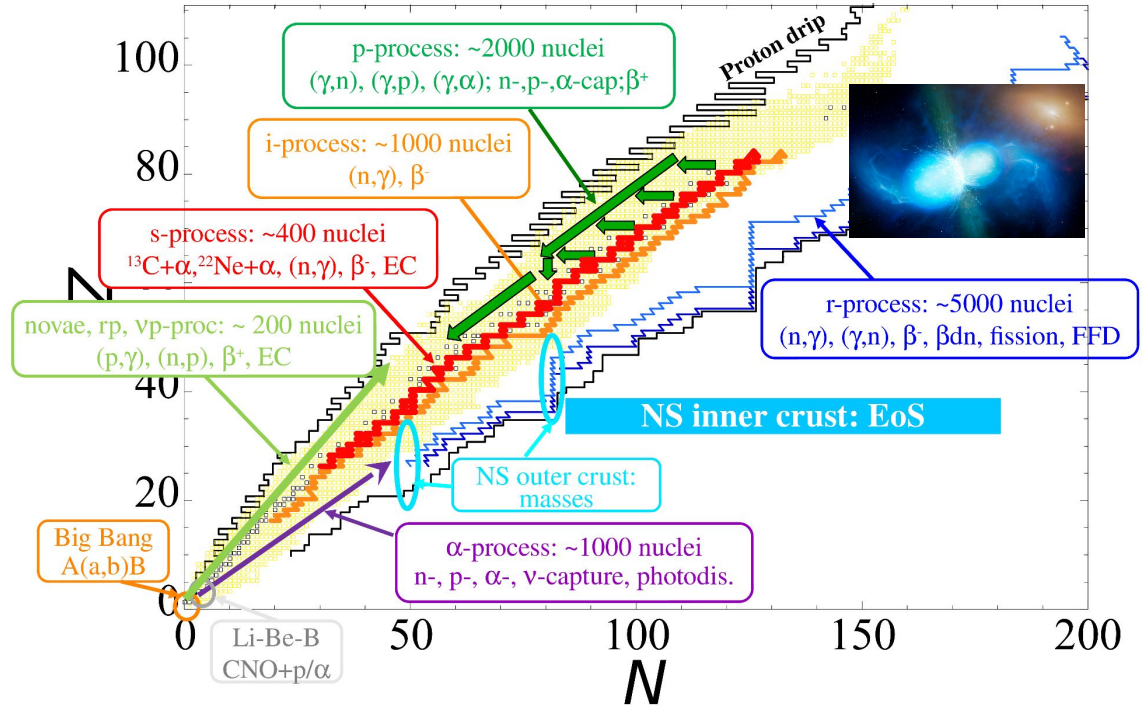
# The nuclear chart and the processes traversing it



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## Extrapolations in

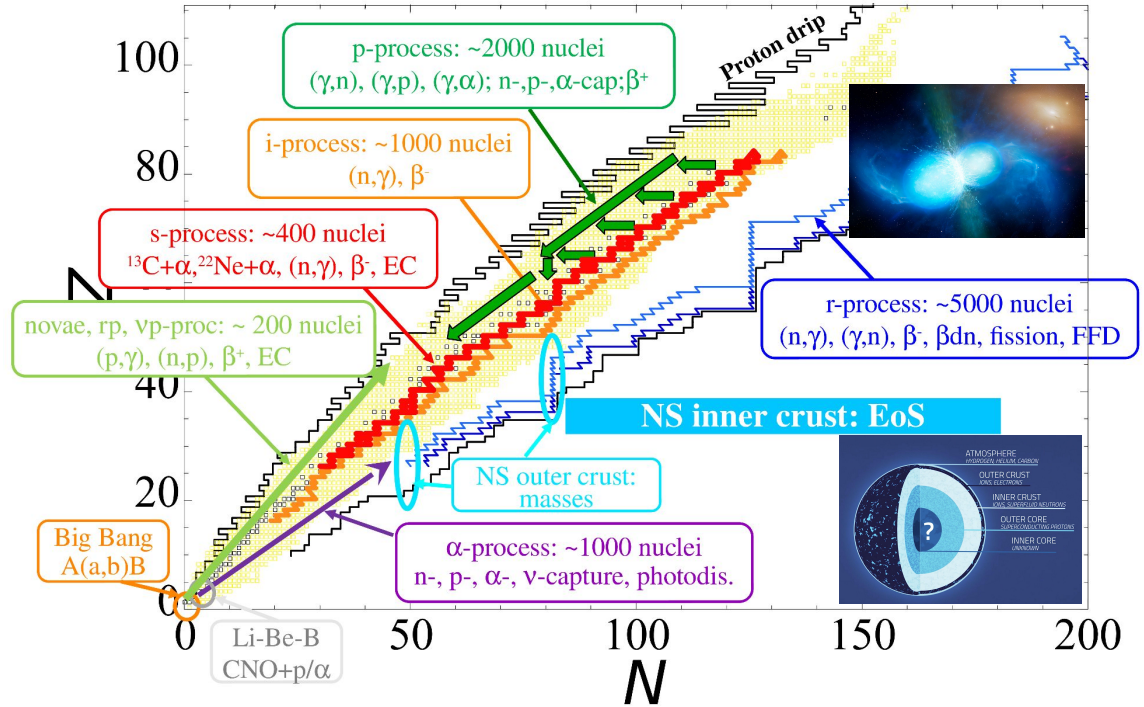
- nucleon number
- energy
- temperature
- density
- .....



# The nuclear chart and the processes traversing it

**Extrapolations in**

- nucleon number
- energy
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- .....



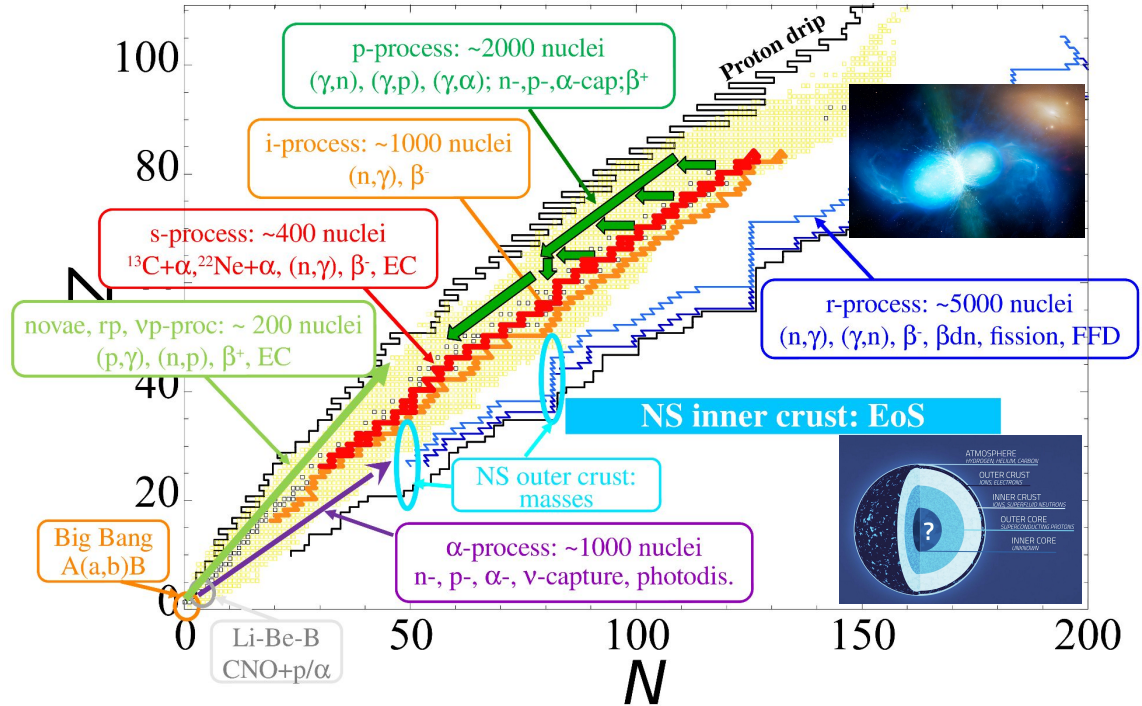
# The nuclear chart and the processes traversing it

## Extrapolations in

- nucleon number
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and all of that for

- ~7000 nuclei
- many reactions



# The nuclear chart and the processes traversing it

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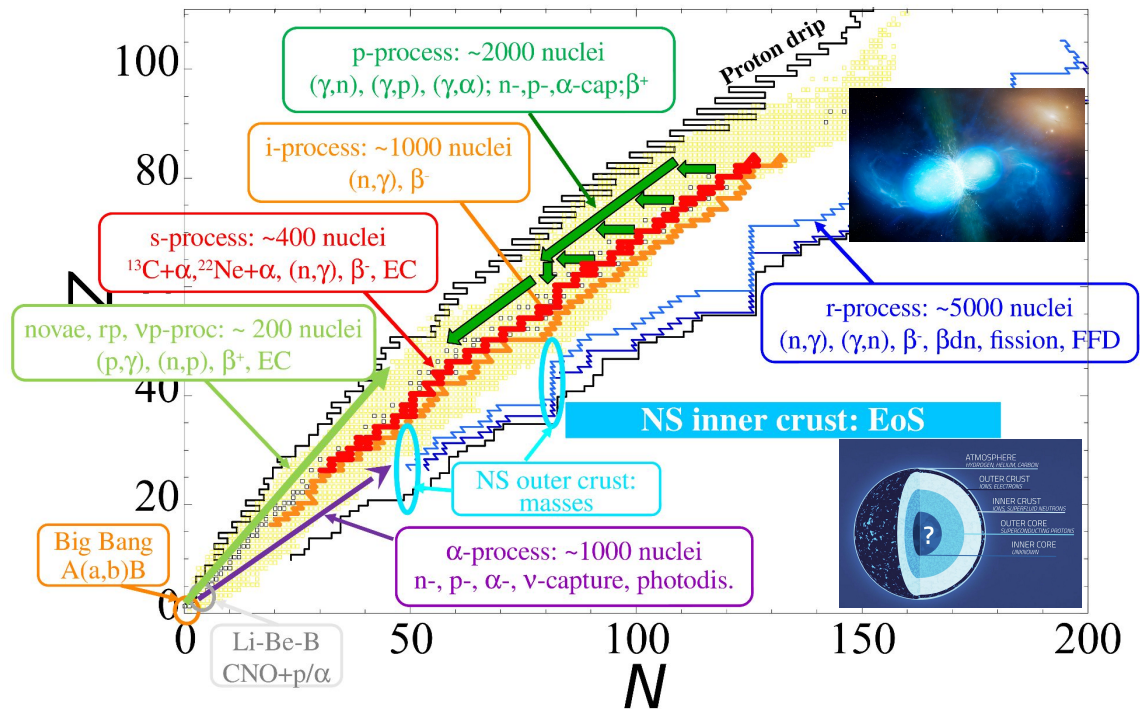
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and all of that for

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what we need is models that should be

1. predictive....
2. but also complete





# Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

$$E \sim \int d^3r \left[ C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$



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Local densities and currents of a wavefunction



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Coupling constants (~ **25** parameters) fitted to data

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## Strong points

- wavefunctions with individual nucleons
- based on “in-medium” N-N interaction
- many observables accessible
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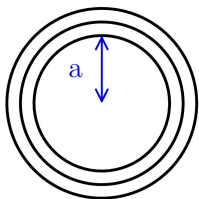
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## How to move forward?

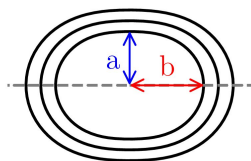
1. search for a “better” EDF form
2. include more experimental information
3. include more physics in the wavefunction

# Large-scale models in 1-2 dimensions

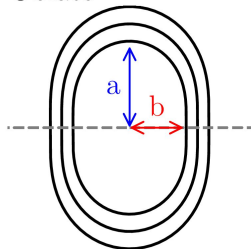
Spherical



Prolate



Oblate



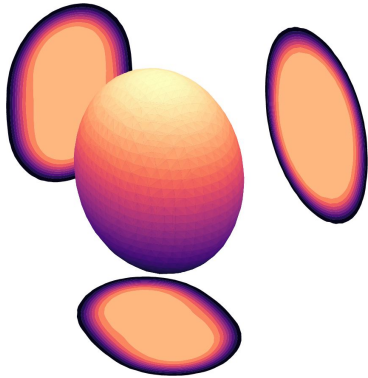
One DOF:  $\beta_{20}$

## Nuclear deformation

- larger variational space
- shape DOF characterized by multipole moments
- capture correlations at modest CPU cost
- intuitive interpretation

# Large-scale models in 1-2-3 dimensions

$\beta_{20}, \beta_{22}$  or  $\beta_2, \gamma$



## Symmetry breaking leads to deformation

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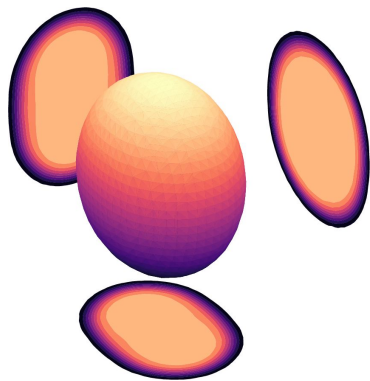
## More general configurations

- triaxial shapes

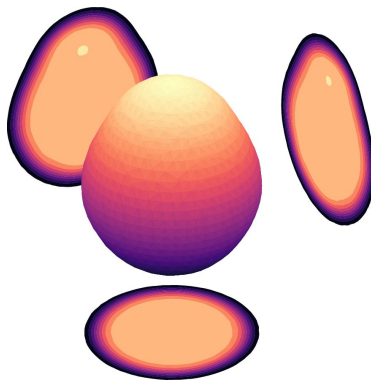


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$\beta_{20}, \beta_{30}$



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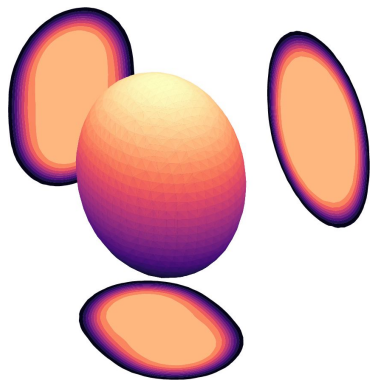
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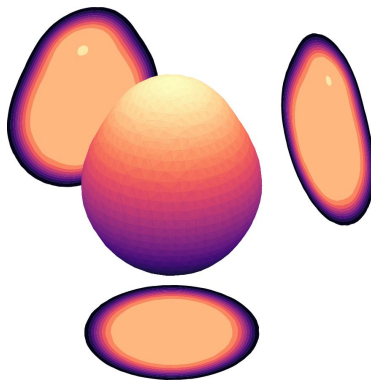
- triaxial shapes
- reflection asymmetry

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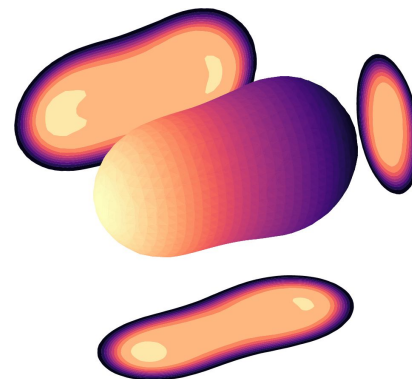
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$\beta_{20}, \beta_{22}$  **and**  $\beta_{30}$



## Symmetry breaking leads to deformation

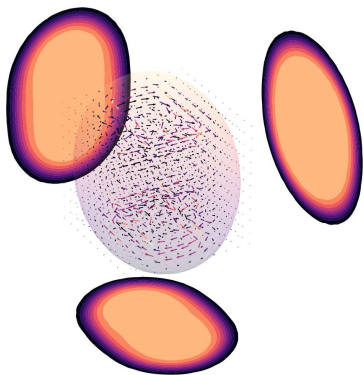
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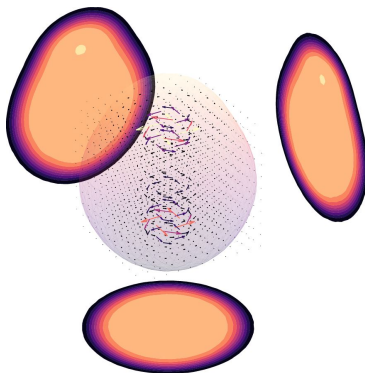
- triaxial shapes
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- elongated shapes

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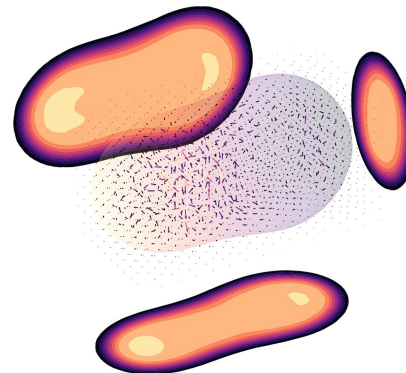
$\beta_{20}, \beta_{22}$  or  $\beta_2, \gamma$



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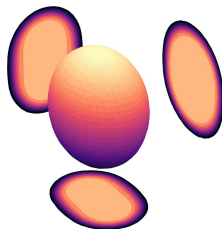
## More general configurations

- triaxial shapes
- reflection asymmetry
- elongated shapes
- spin densities and currents

# Brussels-Skyrme-on-a-Grid: BSkG

## BSkG1 (2021)

- fitted to 2457 masses
- fitted to 884 charge radii
- includes triaxial deformation



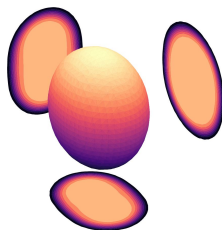
**BSkG1:** G. Scamps et al., EPJA **57**, 333 (2021).  
**BSkG2:** W. Ryssens et al., EPJA **58**, 246 (2022).  
W. Ryssens et al., EPJA **59**, 96 (2023).  
**BSkG3:** G. Grams et al., in preparation.

Rms $\sigma$	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741		
Radii [fm]	0.024		

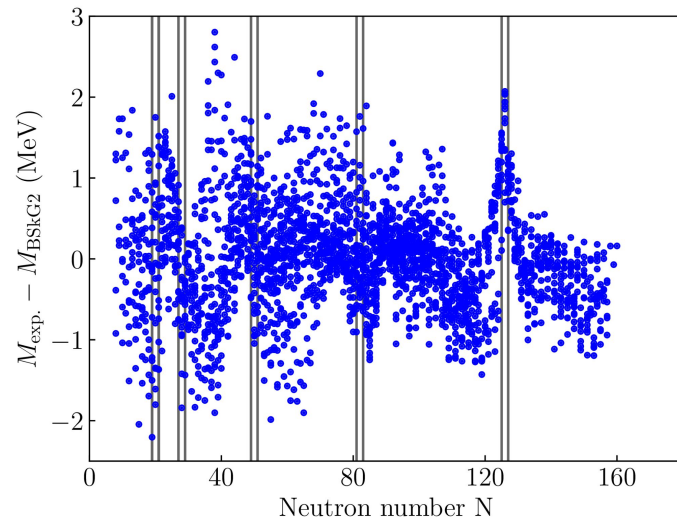
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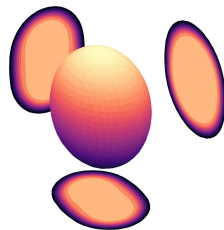
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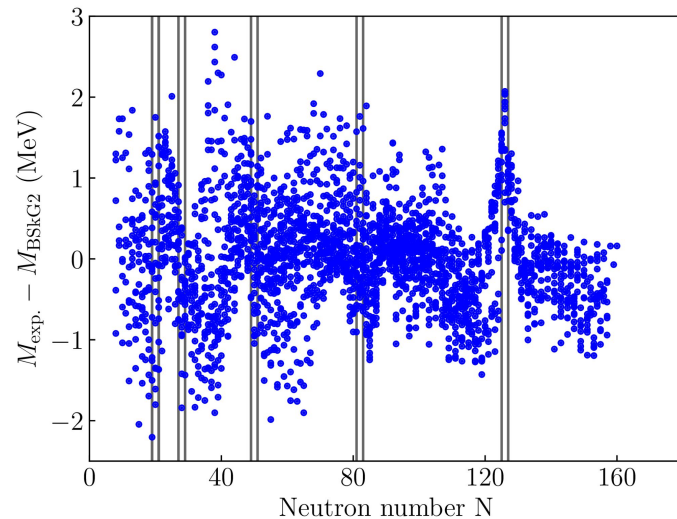
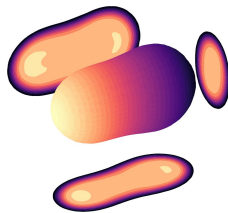
## BSkG1 (2021)

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## BSkG2 (2022)

- fitted to 45 fission barriers
- includes spins, currents,...



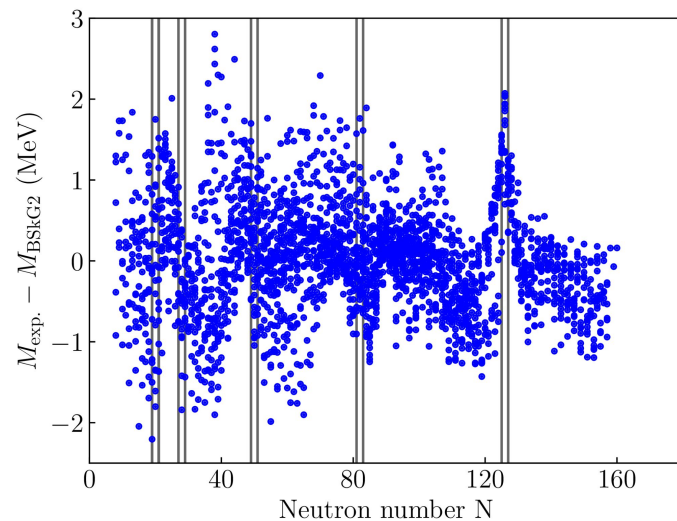
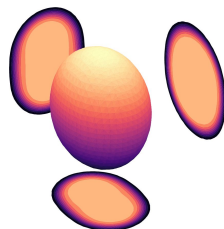
Rms $\sigma$	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741	0.678	
Radii [fm]	0.024	0.027	
Prim. barriers [MeV]	0.88	0.44	
Secun. barriers [MeV]	0.87	0.47	
Fission isomers [MeV]	1.0	0.49	

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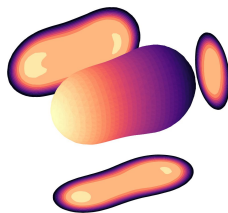
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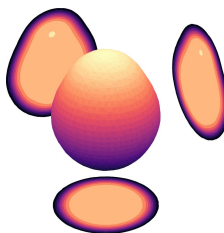
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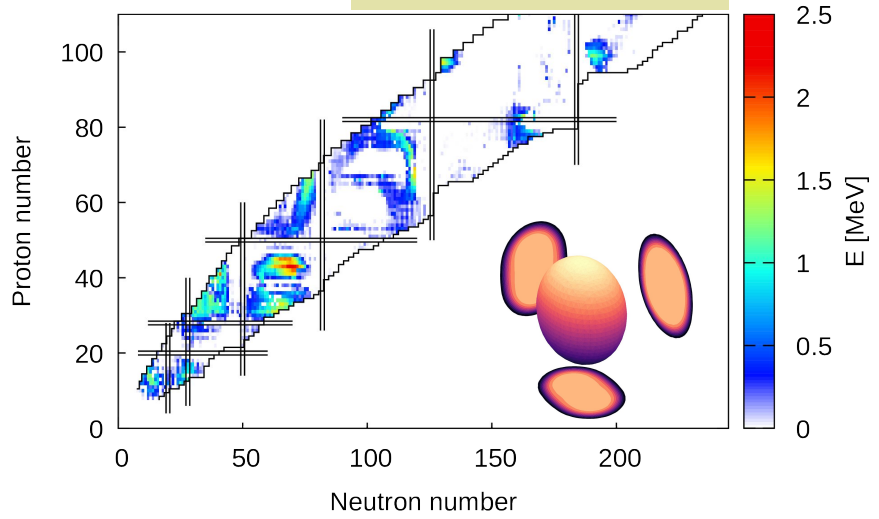
- larger max. neutron star mass
- includes octupole deformation



Rms $\sigma$	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741	0.678	0.631
Radii [fm]	0.024	0.027	0.024
Prim. barriers [MeV]	0.88	0.44	0.33
Secun. barriers [MeV]	0.87	0.47	0.51
Fission isomers [MeV]	1.0	0.49	0.34
Max. NS mass [ $M_{\odot}$ ]	1.8	1.8	2.3

# Masses

G. Scamps et al., EPJA 57, 333 (2021).

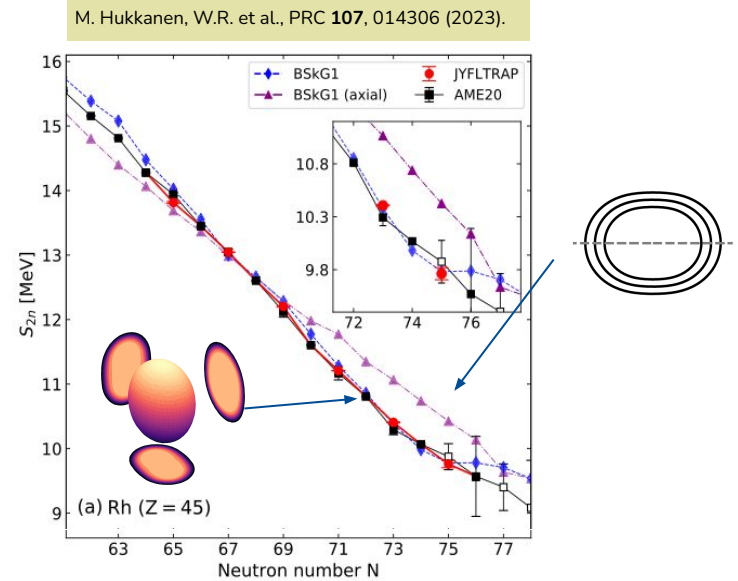
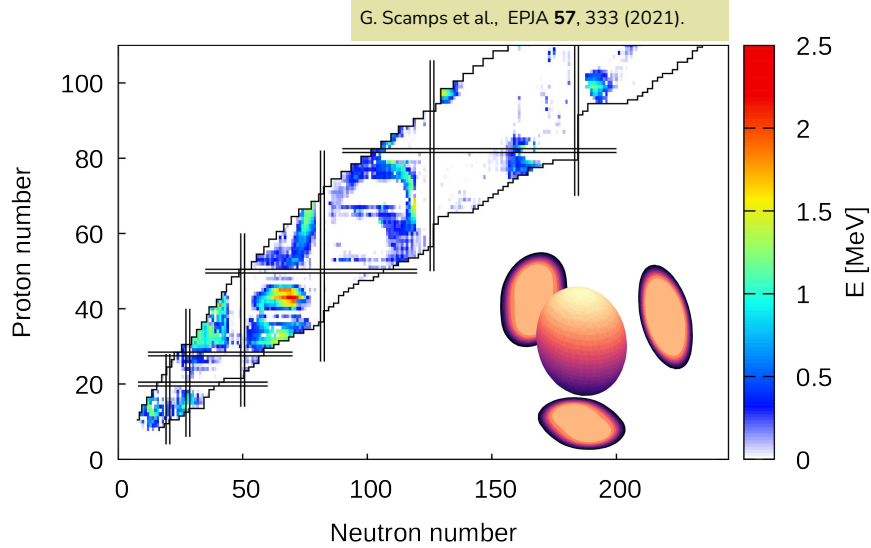


## Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near  $Z \sim 44$



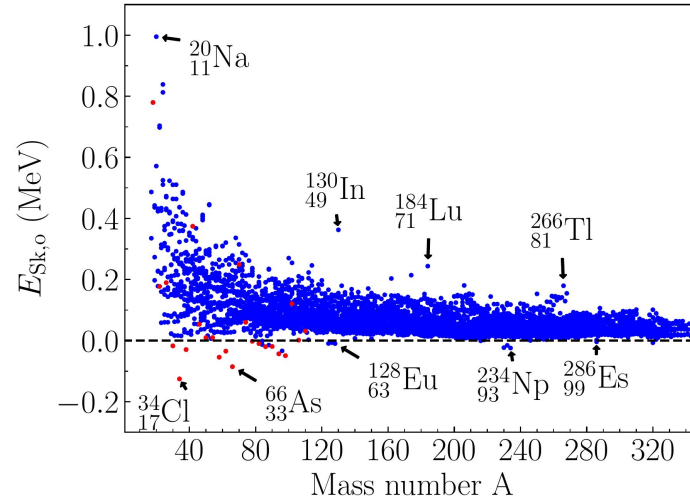
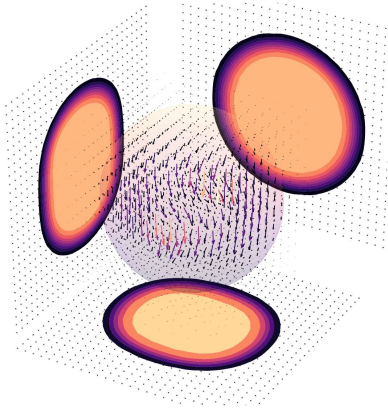
# Masses



## Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near  $Z \sim 44$
- does help reproduce trends, e.g. Rh

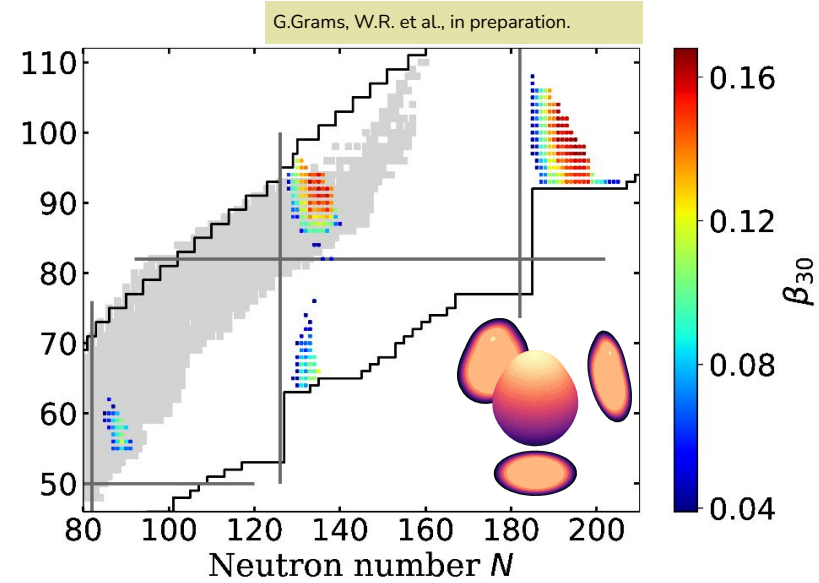
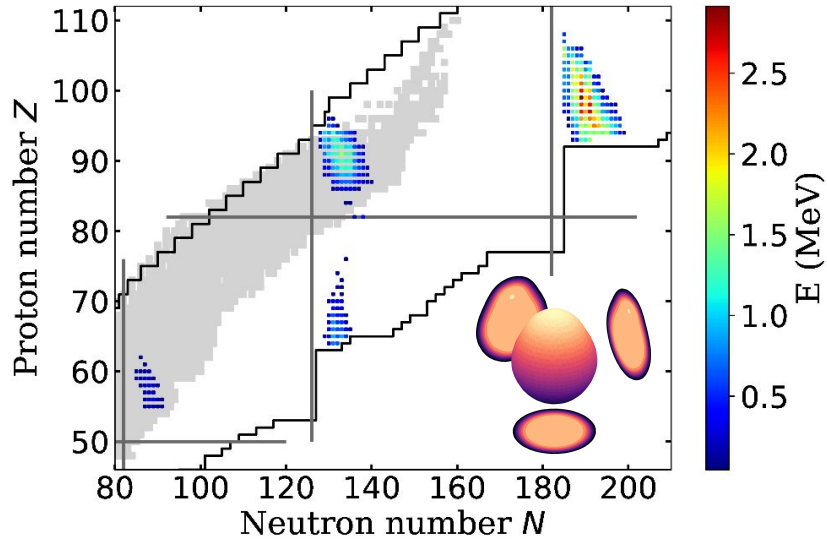
# Masses



## Time-odd terms

- small impact on the masses
- globally repulsive
- first time checked on this scale!
- first step towards other observables

# Masses



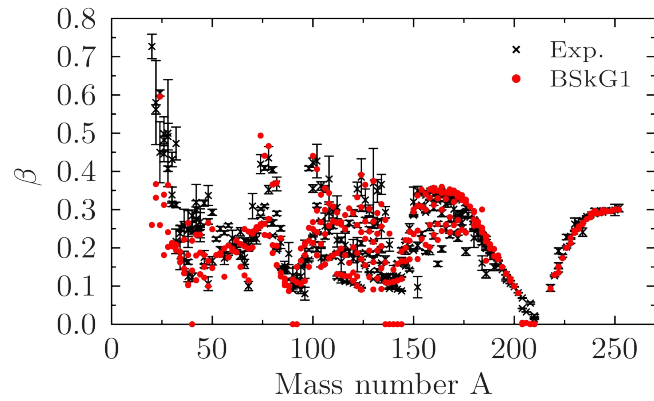
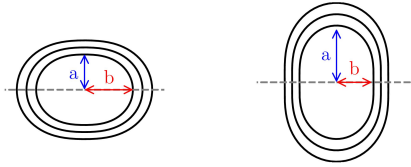
## Reflection asymmetry

- small number of known nuclei affected
- Near  $N=184$ :
  - large effect up to 3 MeV
  - dripline modified
  - fission properties modified

# Deformations

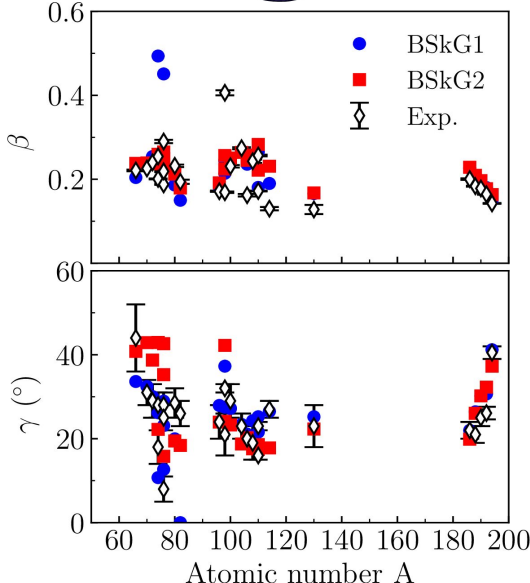
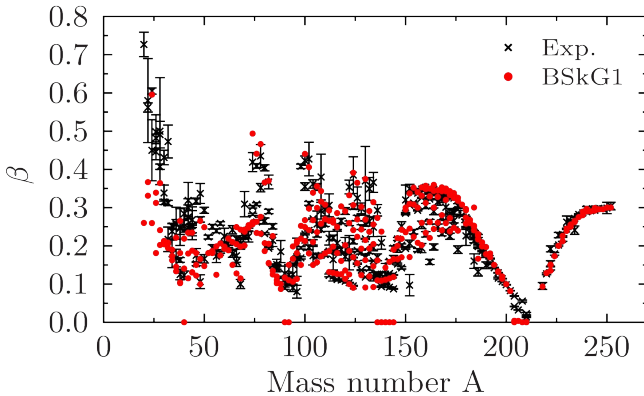
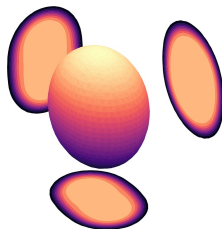
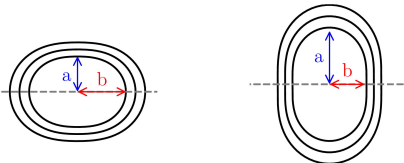
# Deformations

“Ordinary” quadrupole deformation



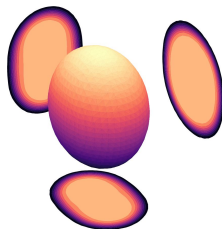
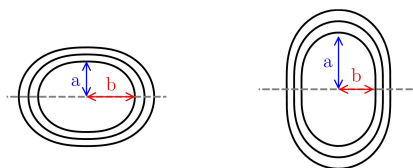
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“Ordinary” quadrupole deformation ... and triaxial deformation ...

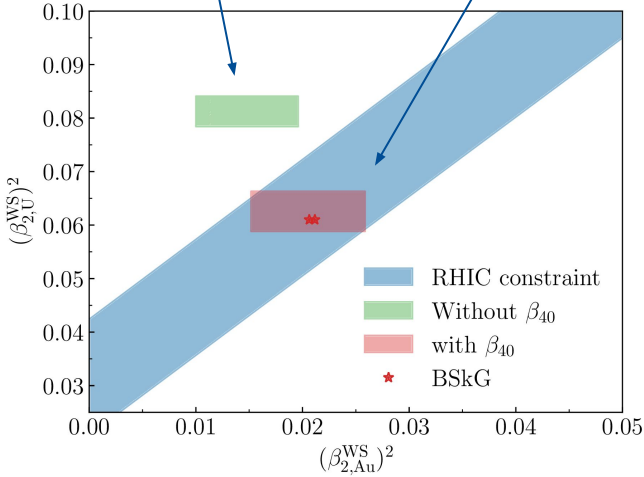
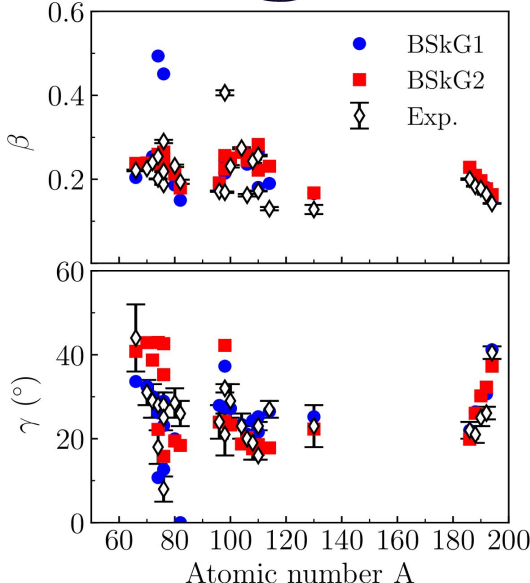
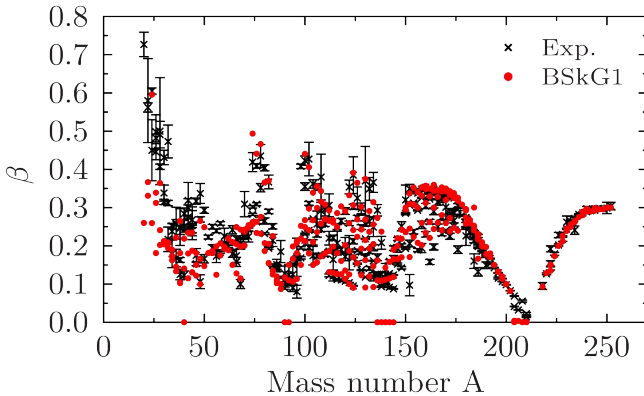


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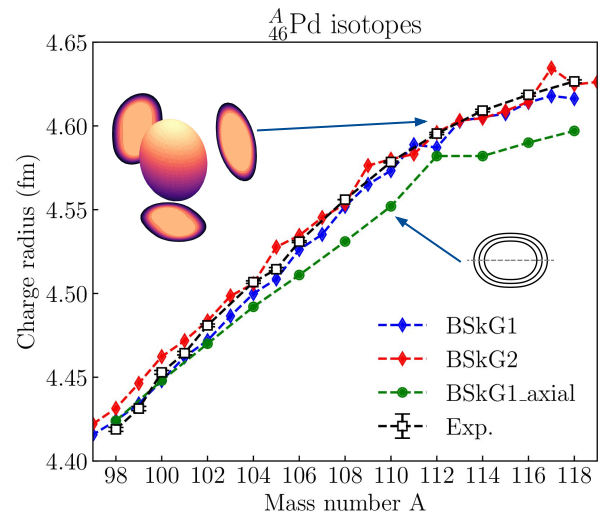


... and even hexadecapole!



# Radii

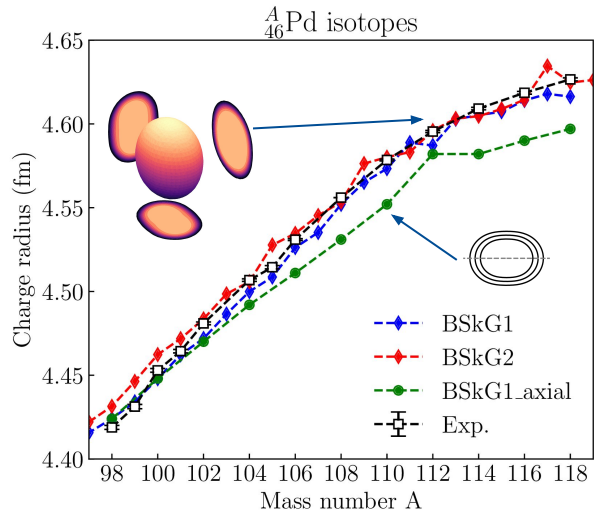
S. Geldhof, PRL **128**, 152501 (2022).



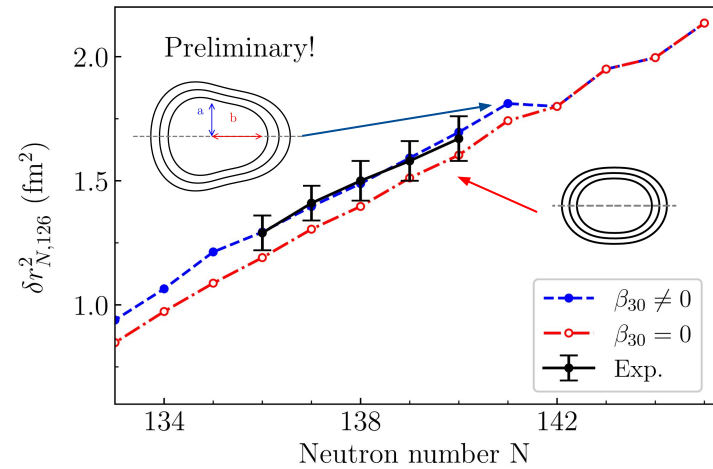


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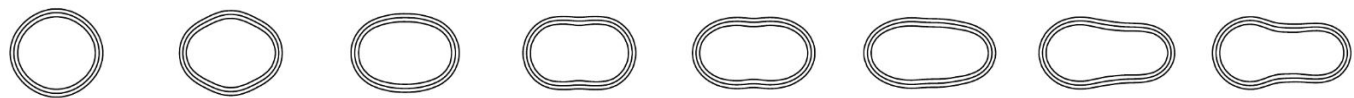
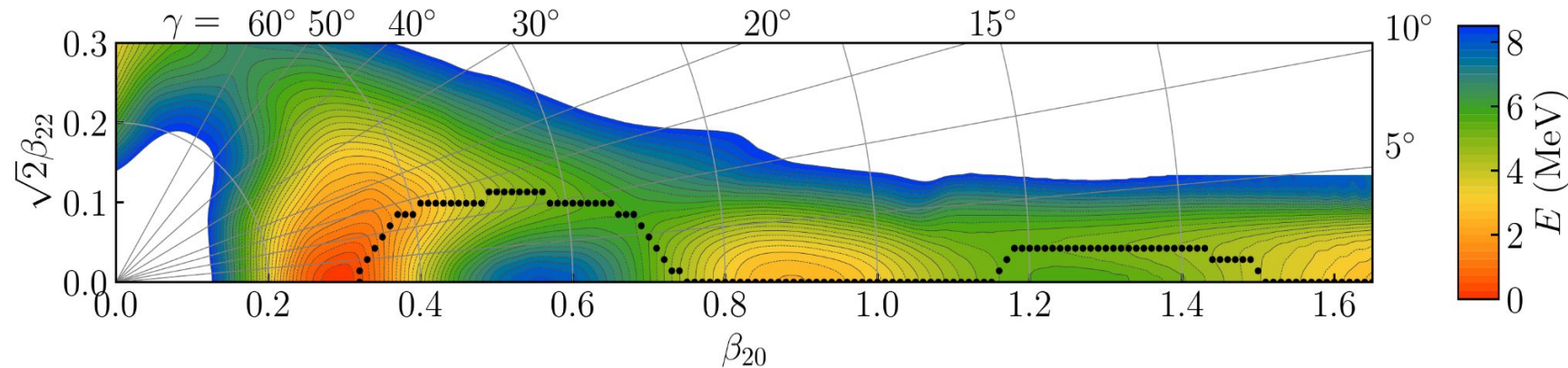
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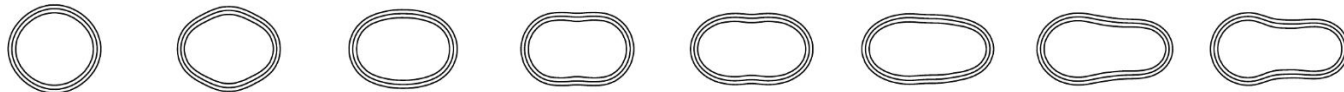
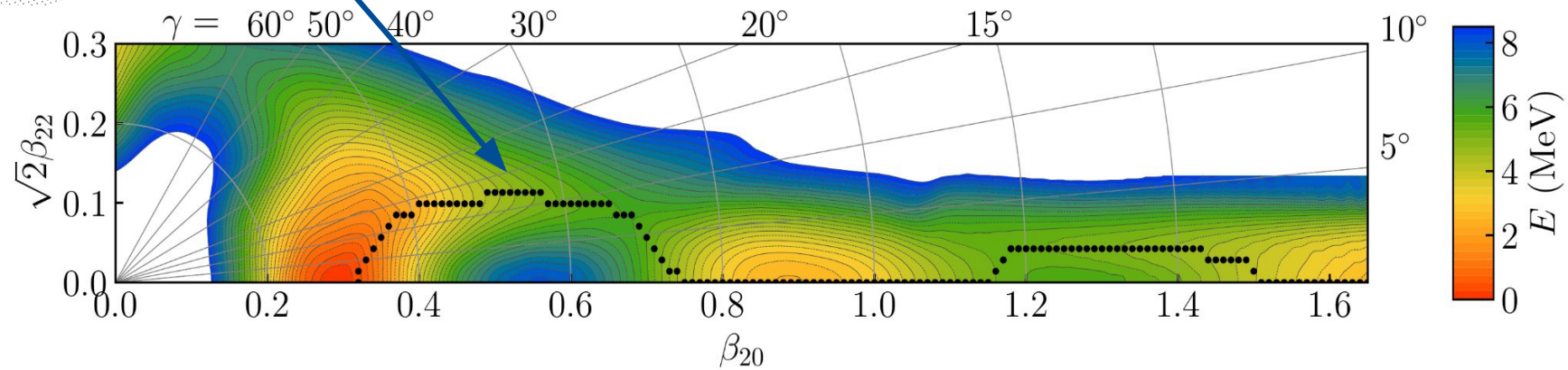
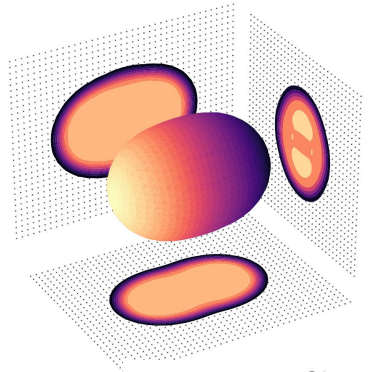
E. Verstraelen, PRC 100, 044321 (2019)



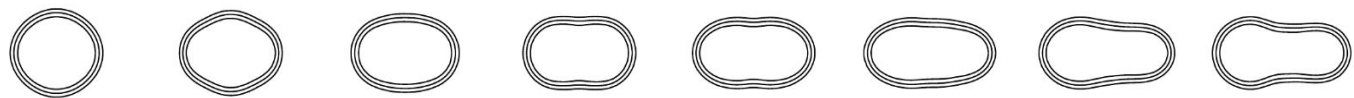
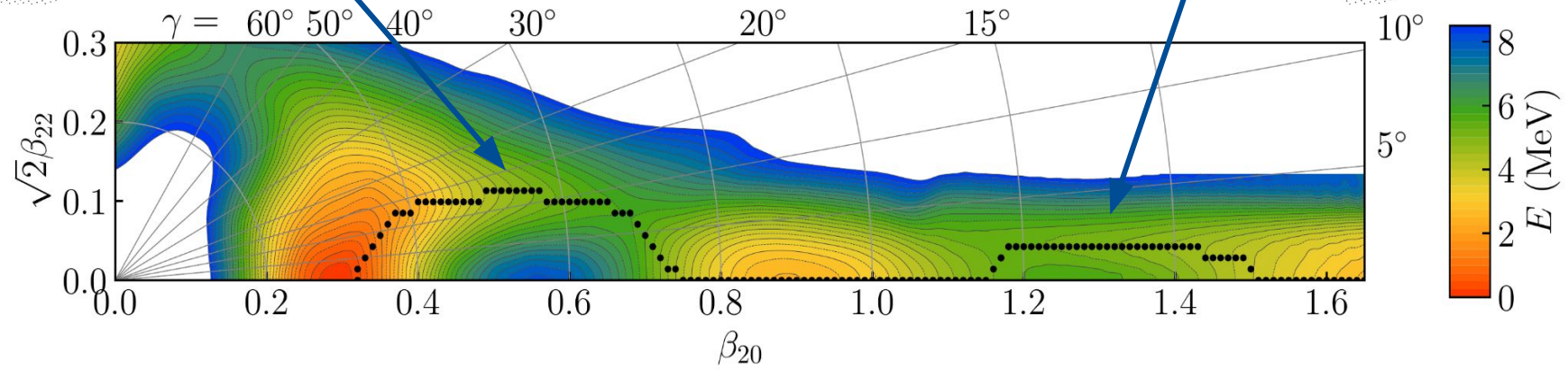
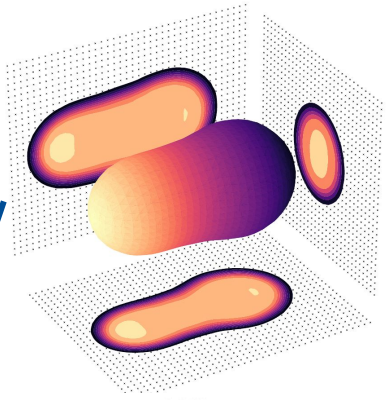
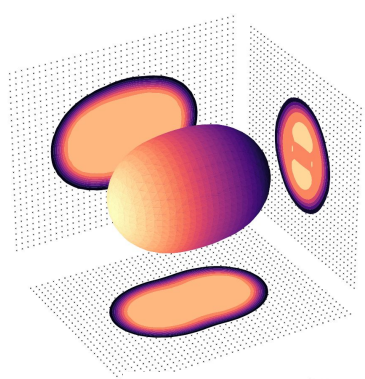
# Fission barriers



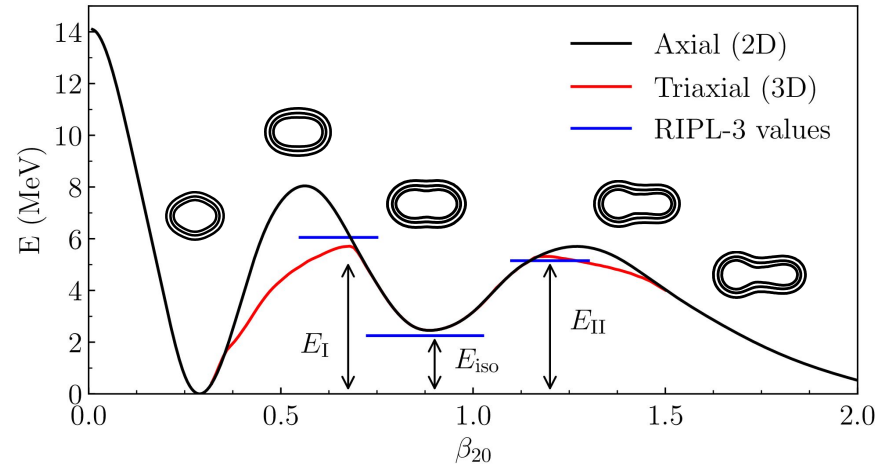
# Fission barriers



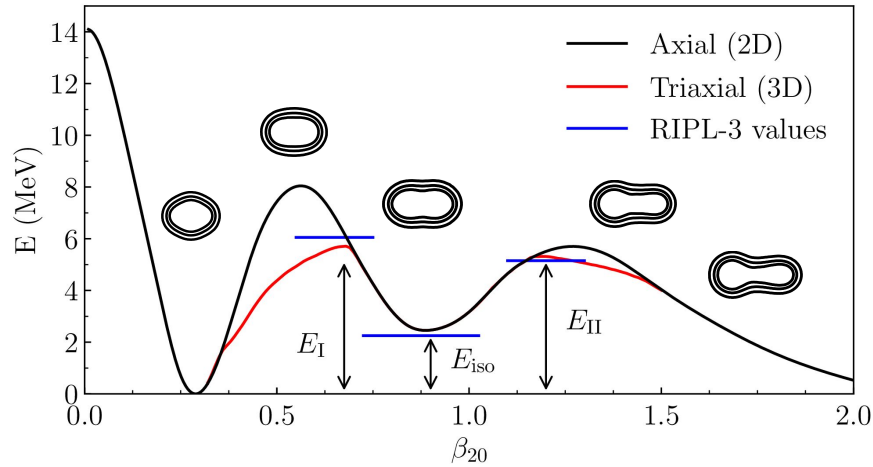
# Fission barriers



# Fission



# Fission



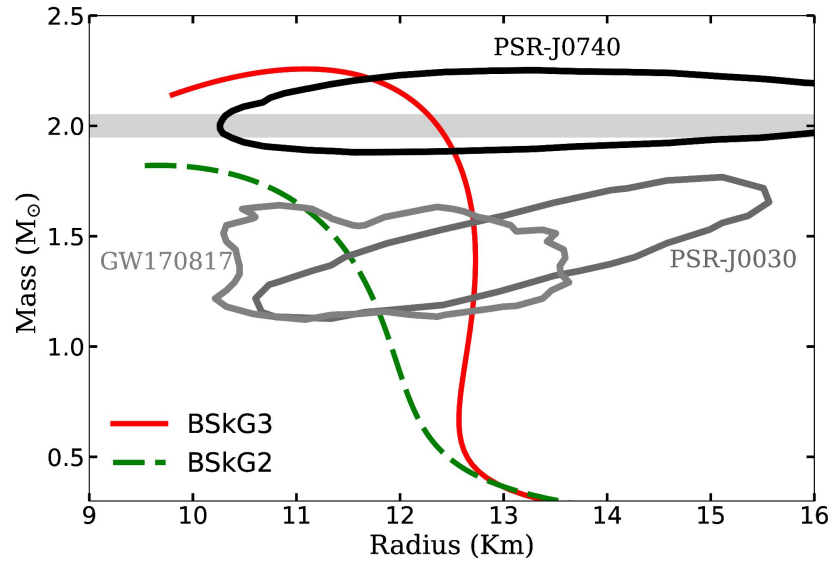
Rms $\sigma$	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741	0.678	0.631
Radii [fm]	0.024	0.027	0.024
Prim. barriers [MeV]	0.88	0.44	0.33
Secun. barriers [MeV]	0.87	0.47	0.51
Fission isomers [MeV]	1.0	0.49	0.34
Max. NS mass [ $M_{\odot}$ ]	1.8	1.8	2.3

## Fission properties of 45 actinide nuclei

- includes odd-A and odd-odds
- **all** inner barriers exploit triaxiality
- **all** outer barriers exploit
  - octupole deformation
  - triaxial deformation

# Neutron stars

G. Grams, W.R. et al., in preparation.

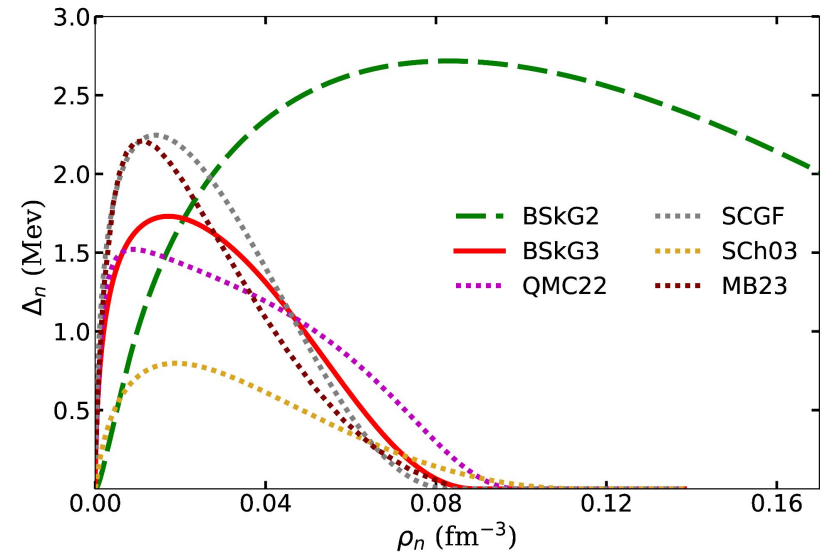
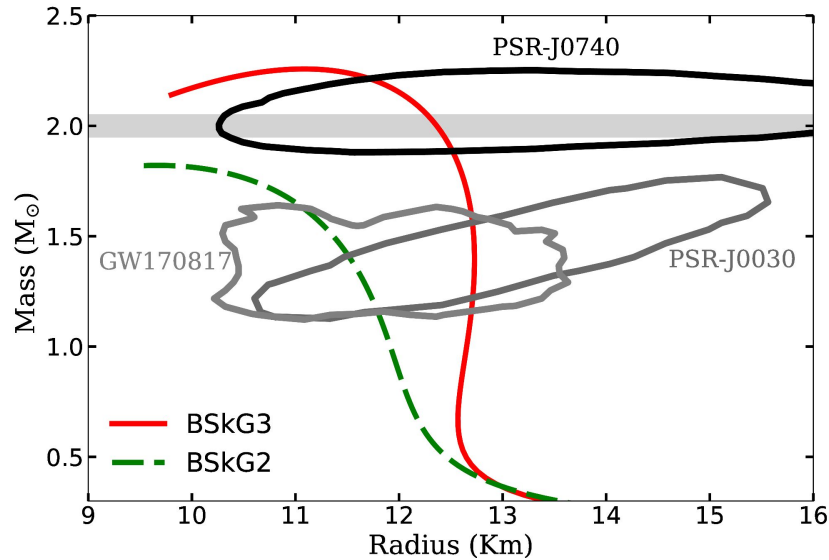


More realistic NS predictions:

- higher maximum mass
  - compatible with NICER
  - compatible with LIGO-VIRGO

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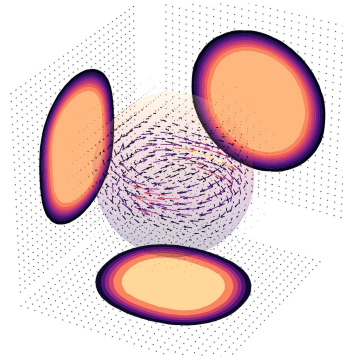


## More realistic NS predictions:

- higher maximum mass
  - compatible with NICER
  - compatible with LIGO-VIRGO
- realistic pairing properties in INM
  - constrained to advanced calculations
- .... but not at the cost of finite nuclei!



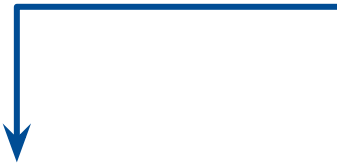
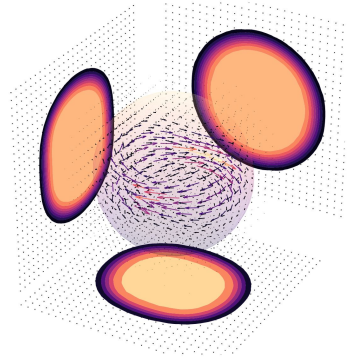
# Additional observables



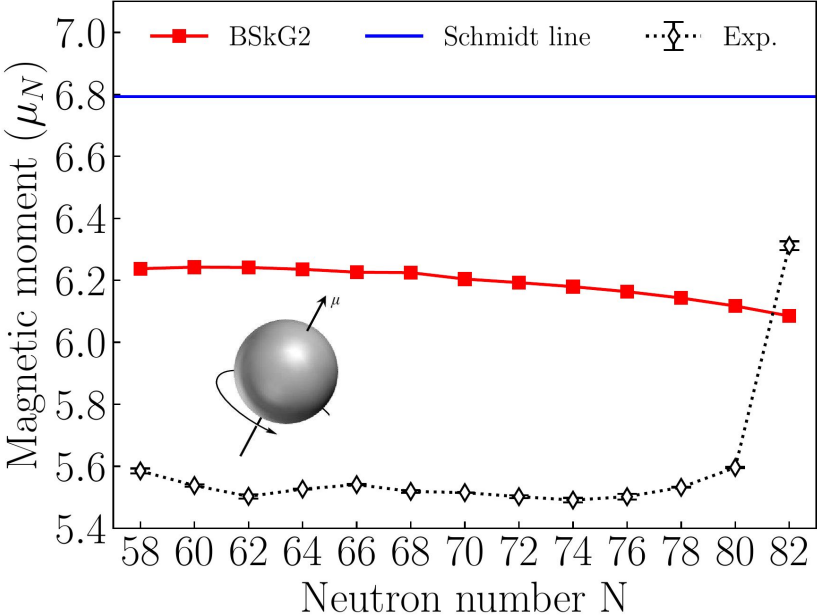
A. R. Vernon et al., *Nature* **607**, **260** (2022),  
J. Eberz et al., *NPA* **464**, 9 (1987).  
J.Y. Zeng et al. *PRC* **50**, 1388 (1994)

# Additional observables

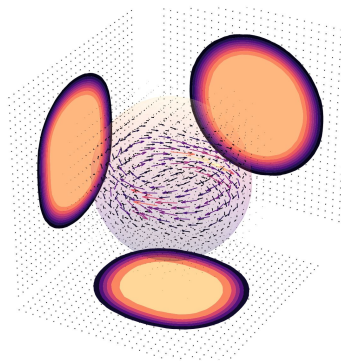
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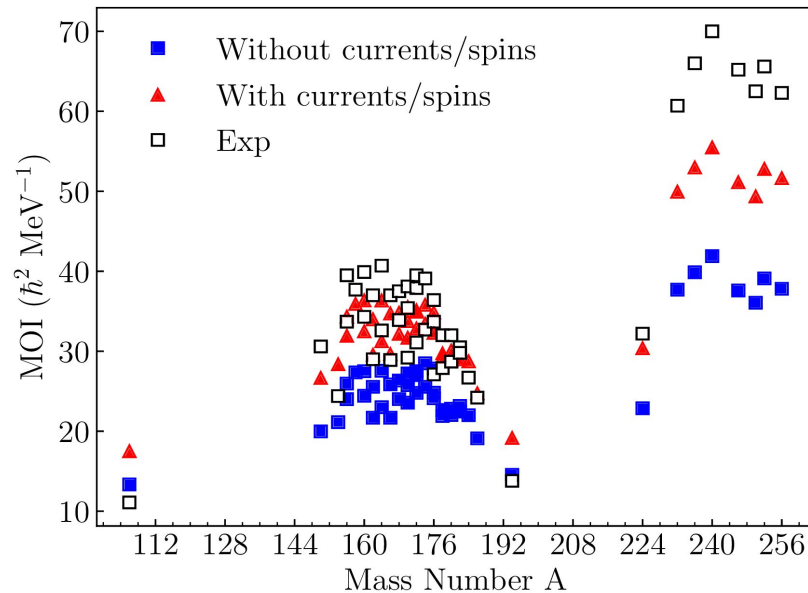
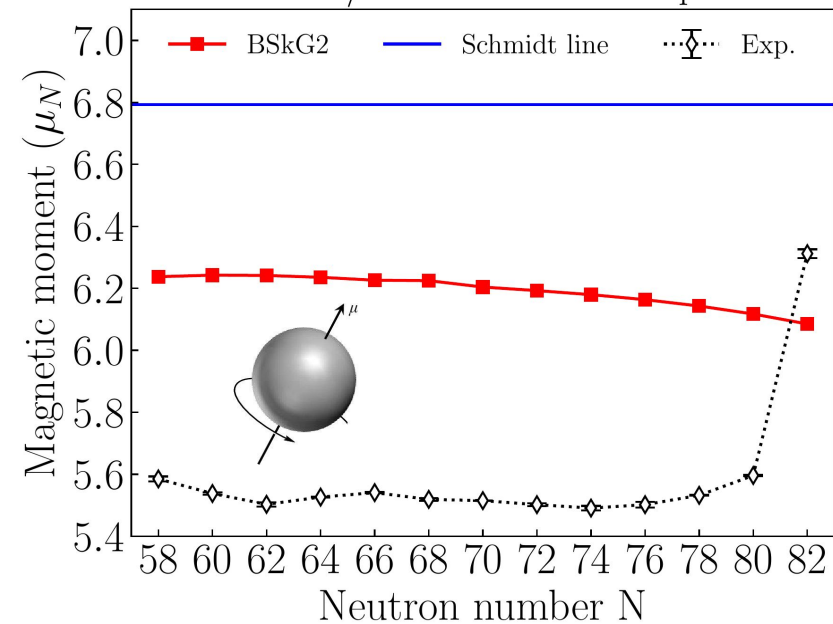
$J^\pi = 9/2^+$  states in In isotopes



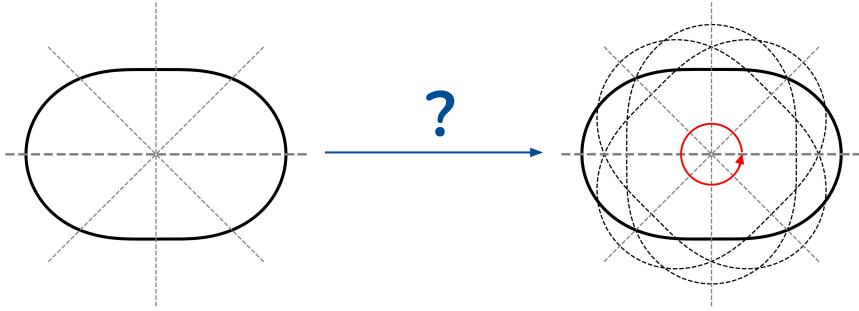
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# Challenges: less phenomenology

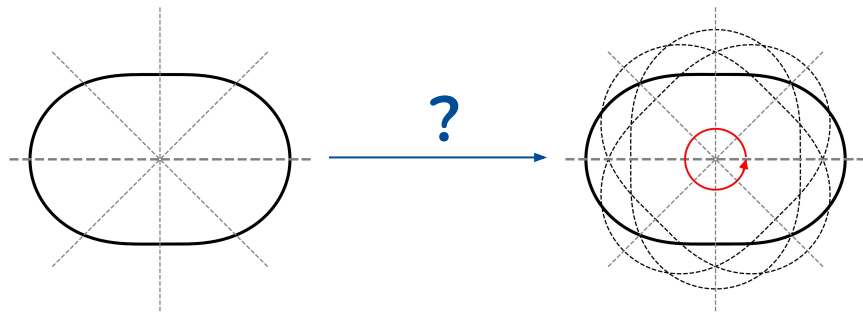


## Leave the mean-field picture behind

- techniques exist
- ... but remain extremely costly

# Challenges: less phenomenology

$$E \sim \int d^3r \left[ C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + ?.. \right]$$



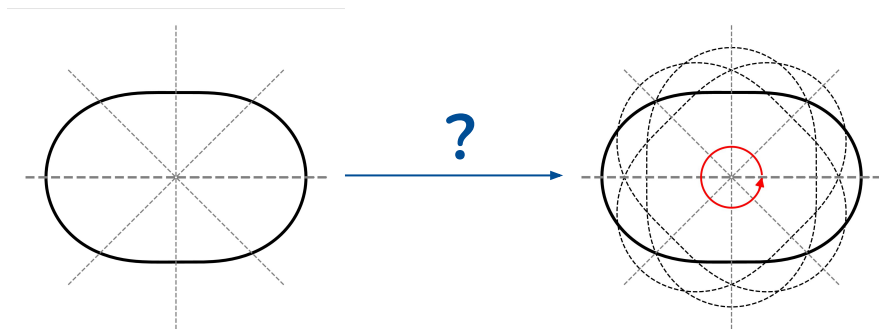
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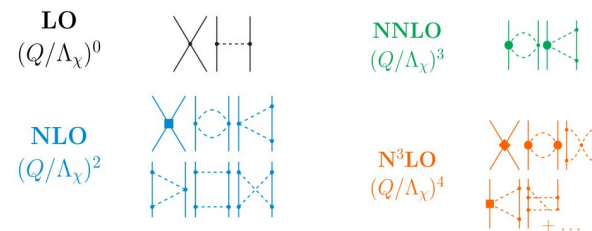
## A “well-founded” functional

- traditional EDF forms are wearing out
- ways for **systematic** construction?
- ... perhaps by linking with ab initio?

# Challenges: less phenomenology



$$E \sim \int d^3r \left[ C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \text{?} \dots \right]$$



Adapted from H. Hergert, Front. Phys. 8:379 (2020).

## Leave the mean-field picture behind

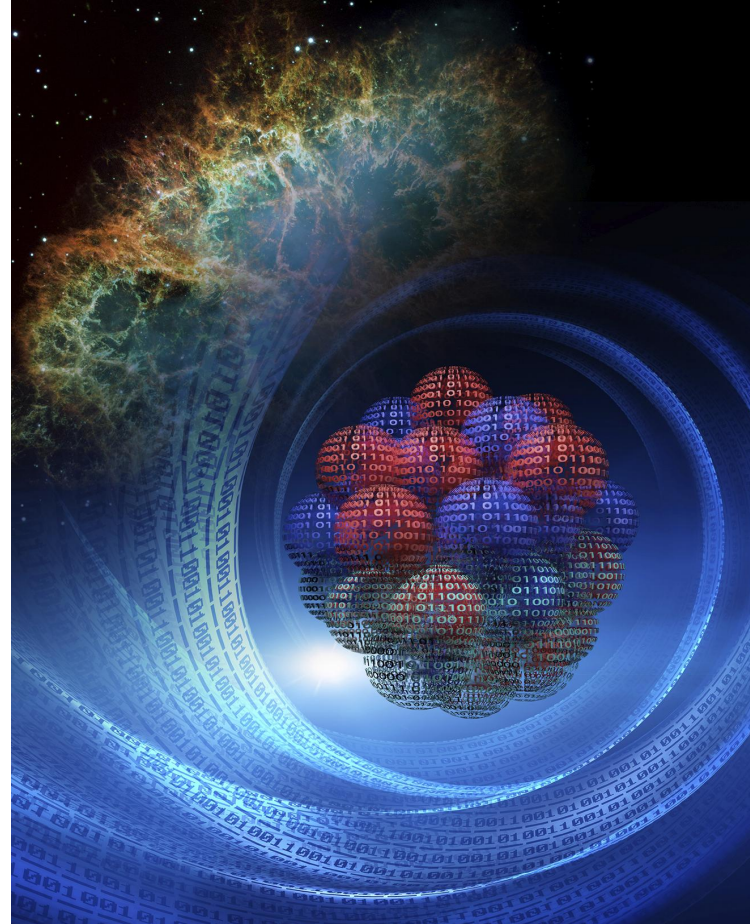
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# Conclusion

We build large-scale, microscopic models for (astro) applications.

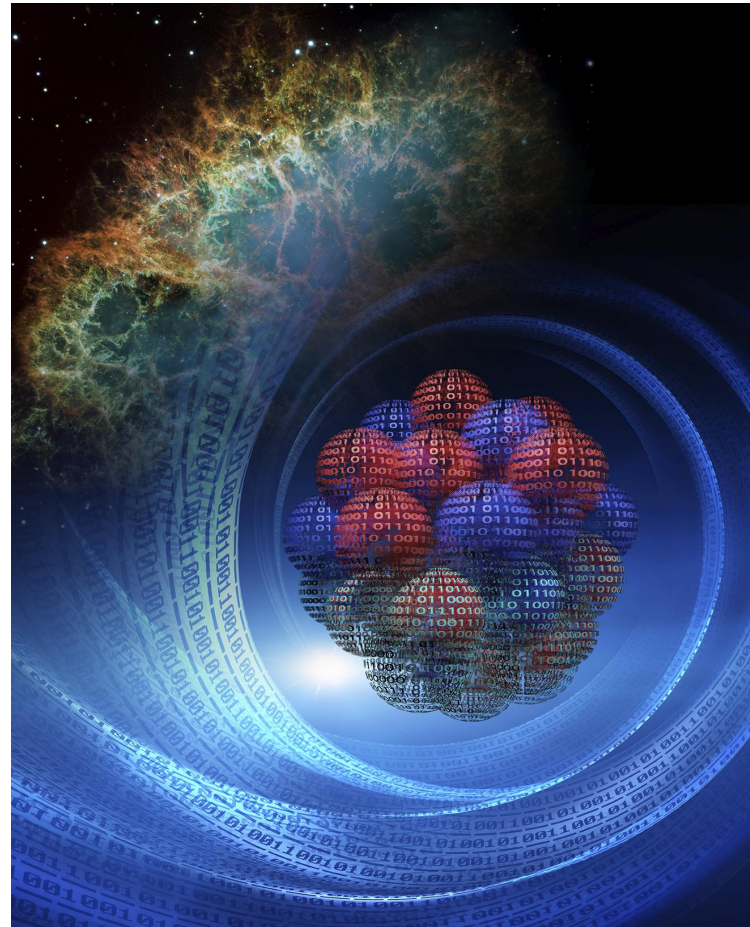


# Conclusion

We build large-scale, microscopic models for (astro) applications.

Large-scale = thousands of nuclei and many observables.

Microscopic = simple wave functions yet complex symmetry breaking.





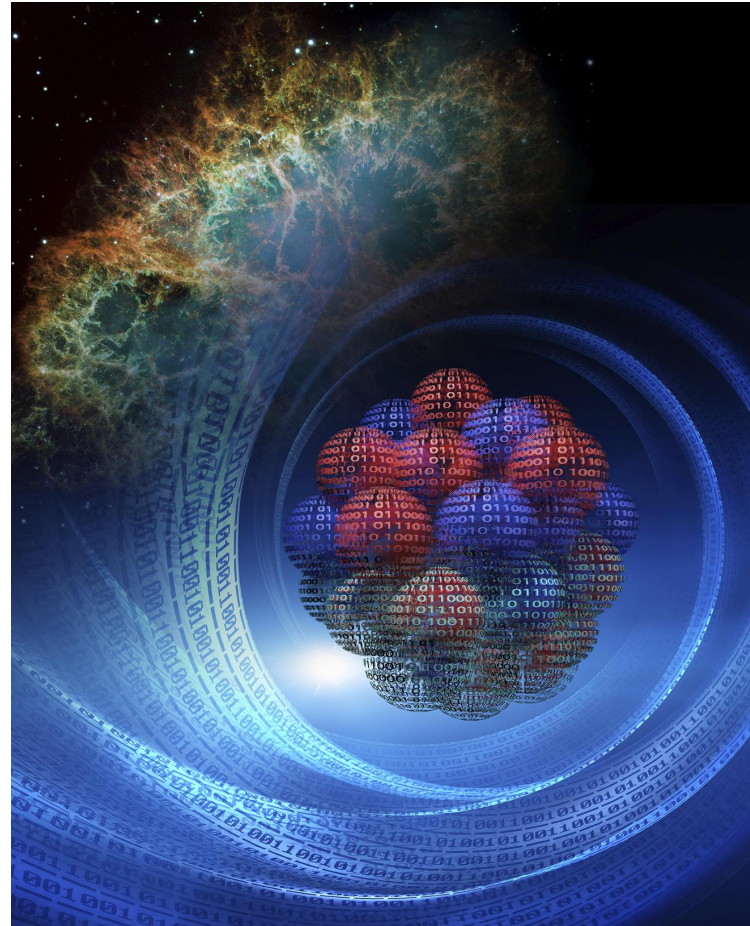
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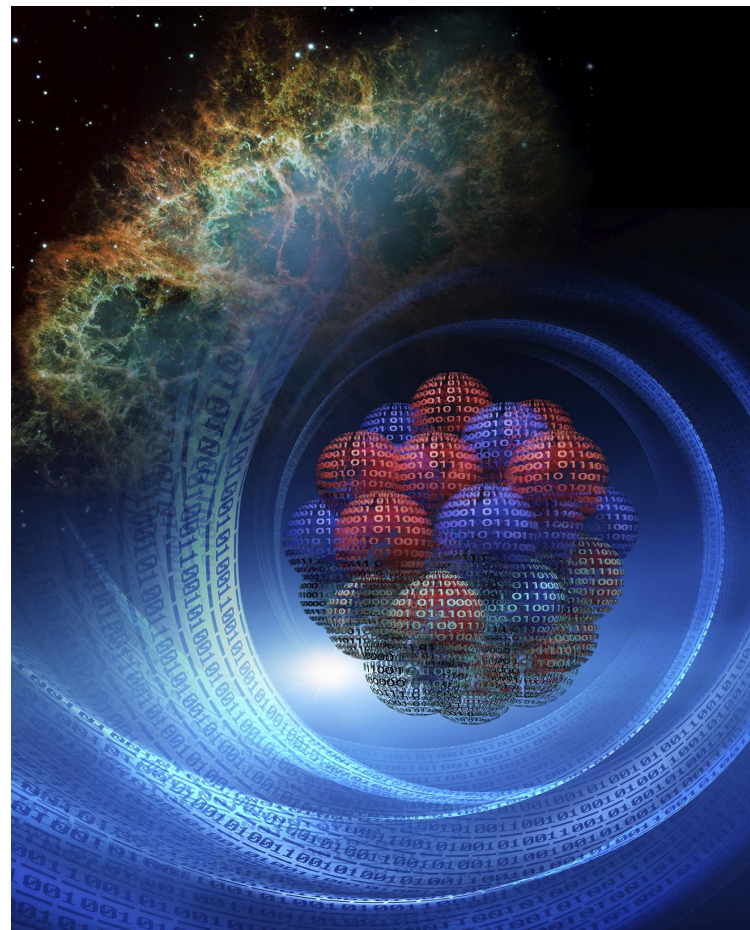
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- global inclusion of
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  - time-reversal breaking
  - octupole deformation
- competitive reproduction of masses and charge radii
- best on the market for **fission** properties
- consistent with astrophysical observations



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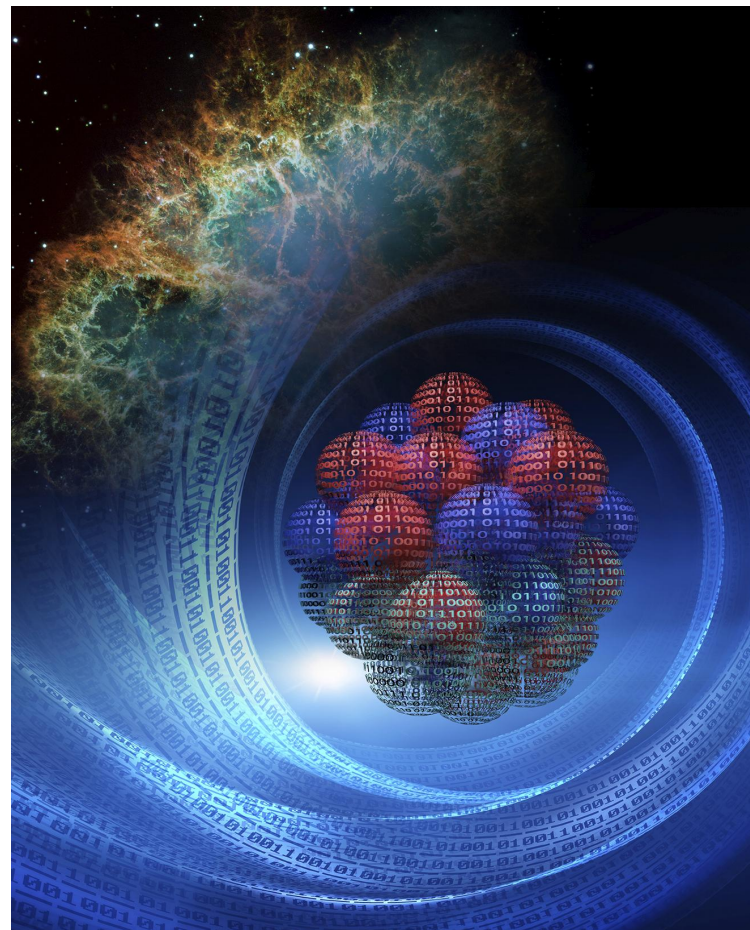
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- consistent with astrophysical observations

Coming up from the Brussels group:

- all BSkG3 data
- detailed study of ground state densities
- large-scale **fission** and **level density** calculations
- unified Equation of State for neutron star applications



# Thank you for...

..... all the wonderful work!



**S. Goriely**

**G. Grams**

N. Chamel

N. Shchepochin



**M. Bender**

J. Bonnard



G. Scamps



M. Hukkanen

M. Stryjczyk

A. Kankainen



P. Ascher

S. Grévy



E. Verstraelen

T. Cocolios

P. Van Duppen



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SEIT 1386

G. Giacalone



B. Schenke

C. Shen



S. Hilaire

..... the computing time!



..... the funding!



..... your attention!

2013



ULB

Mean-field  
description of  
rotating  
ellipsoids in  
the rare earth  
region

Wouter  
Ryssens

Mean-Field  
Symmetries

Rare-Earths:  
 $3d^5 4f^7$

Progress and  
Outlook

## Mean-field description of rotating ellipsoids in the rare earth region

Wouter Ryssens

ULB

2 December  
Brussels

2013



**ULB**

Mean-field description of rotating ellipsoids in the rare earth region  
Wouter Ryssens

Mean-Field Symmetries  
Rare-Earths:  $1.55 E_r$   
Progress and Outlook

## Mean-field description of rotating ellipsoids in the rare earth region

Wouter Ryssens

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Brussels

Navigation icons: back, forward, search, etc.

2023



WOUTER RYSSENS

A visualization of microscopic models of nuclear structure, showing a central cluster of red and blue spheres surrounded by a blue, glowing, circular structure with intricate patterns.

Microscopic models of nuclear structure: from dripline to dripline

Wouter Ryssens, G. Grams, M. Bender and S. Goriely

3th of July 2023

Andy Sparrow, CRNL

Navigation icons: ID, Twitter, website, wryssens.com, wryssens@ulb.be

ULB fnls

Thank you for **a decade** of support!

2013



ULB

Mean-field description of rotating ellipsoids in the rare earth region  
Wouter Ryssens

Mean-Field Symmetries

Rare-Earths:  $3.5^2 E_r$

Progress and Outlook

Mean-field description of rotating ellipsoids in the rare earth region

Wouter Ryssens

ULB

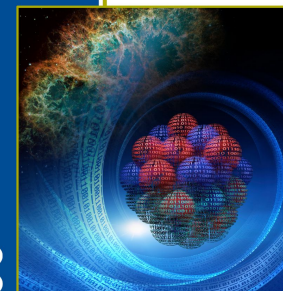
2 December  
Brussels



2023



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Microscopic models of nuclear structure: from dripline to dripline

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wryssens.com

wryssens@ulb.be

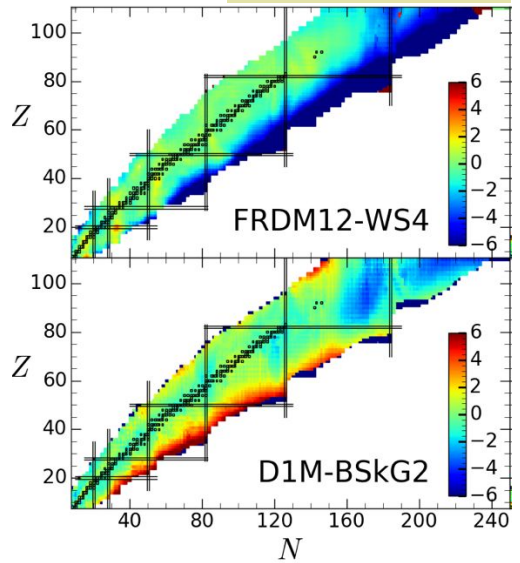


Bonus!

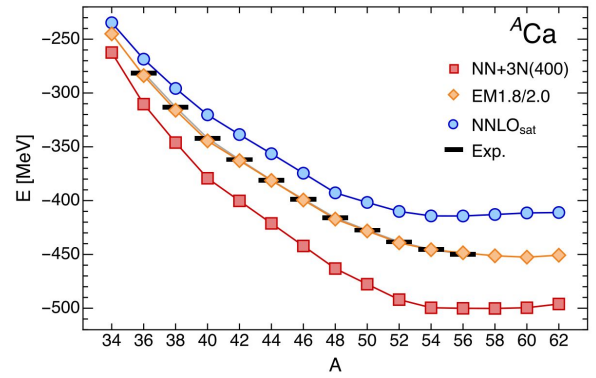
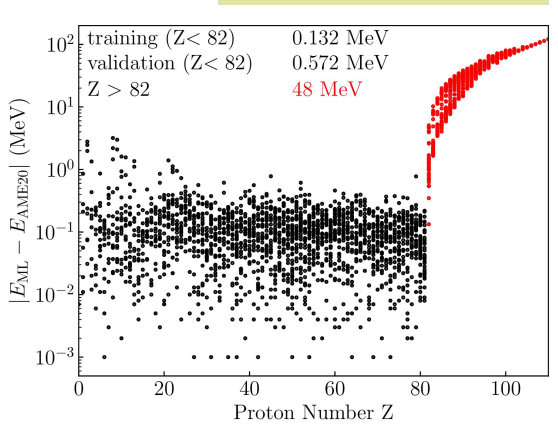


# Interlude: why do we do these complex things?

S. Goriely, EPJA 59, 16 (2023).



G. Grams, W.R. et al., in preparation



## Mic-mac approaches?

- ✓ competitive in rms
- ✓ multiple observables
- ✗ comparatively unstable
- ✗ no link mic. <-> mac.

## Machine learning?

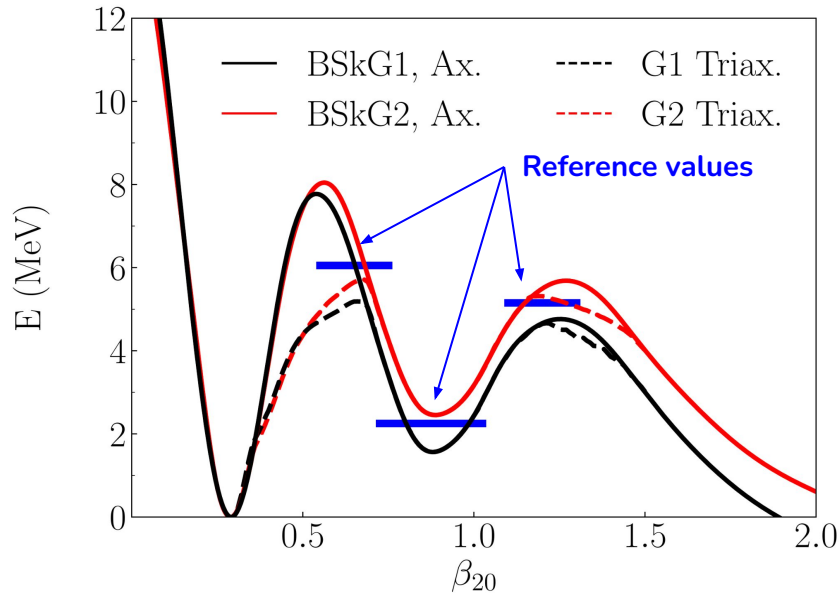
- ✓ absolute champion in rms
- ✓ ridiculously easy
- ✗ thousands (?) of parameters
- ✗ single observable

## Ab Initio?

- ✓ error quantification
- ✓ “truly” microscopic
- ✓ multiple observables
- ✗ infeasible at scale (for now)
- ✗ not competitive on rms (for now)

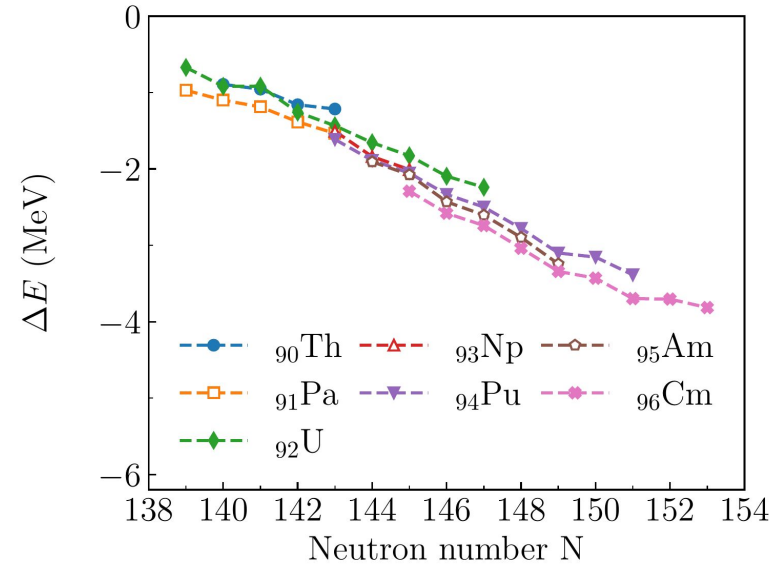
# Triaxiality has a **large** effect on barriers

Reference values from R. Capote et al., Nuclear Data Sheets **110**, 3107 (2009).



## Triaxial deformation for $^{240}\text{Pu}$

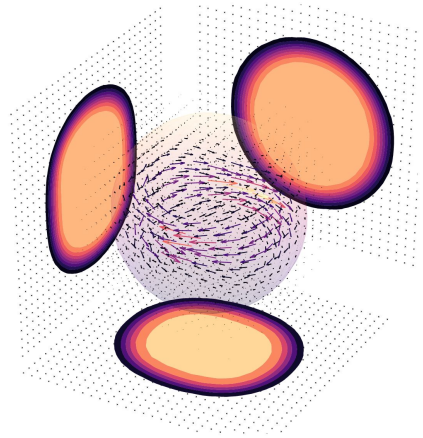
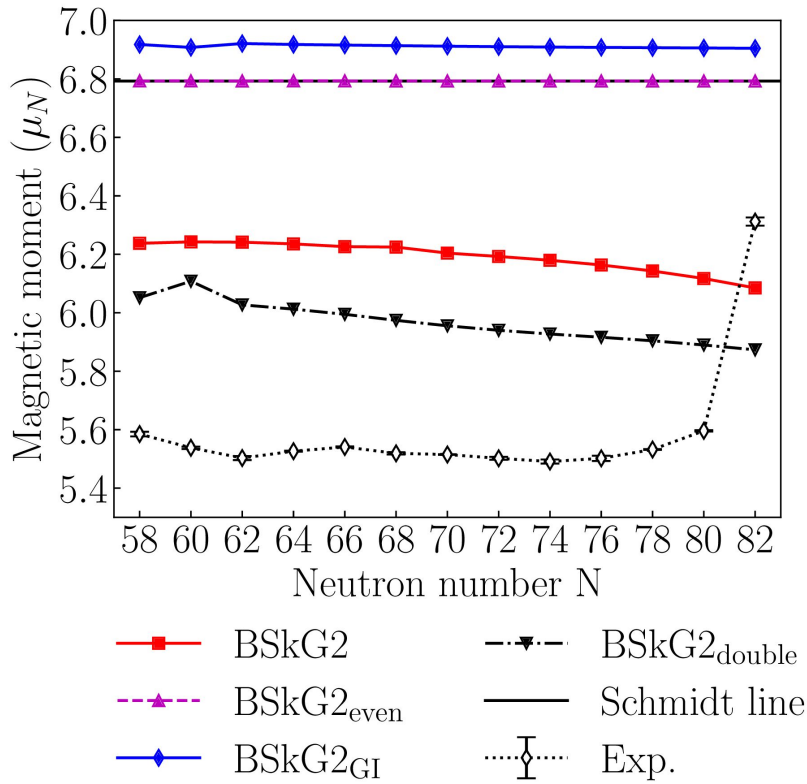
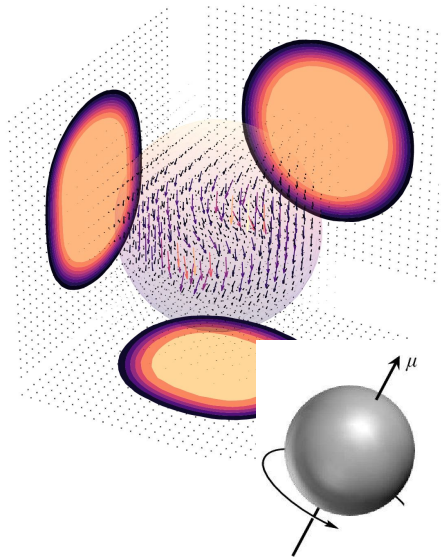
- Large effect on inner barrier
- No effect on isomers
- Modest effect on outer barrier



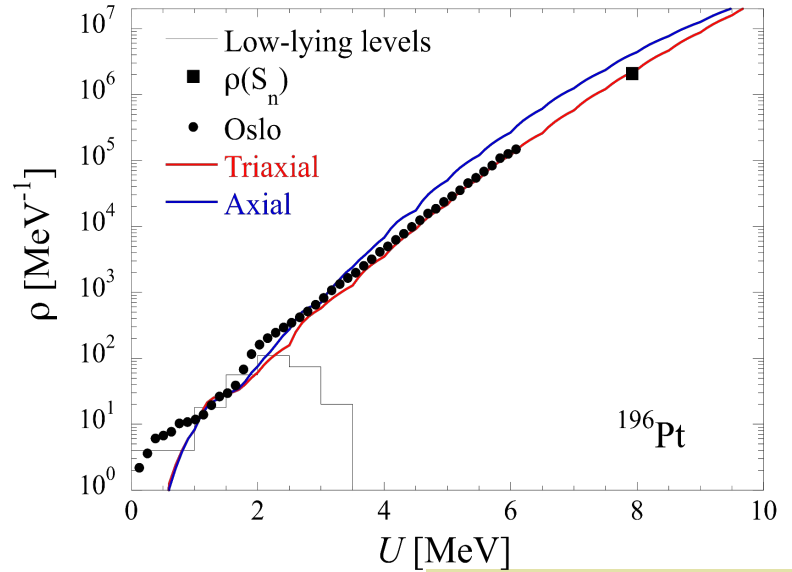
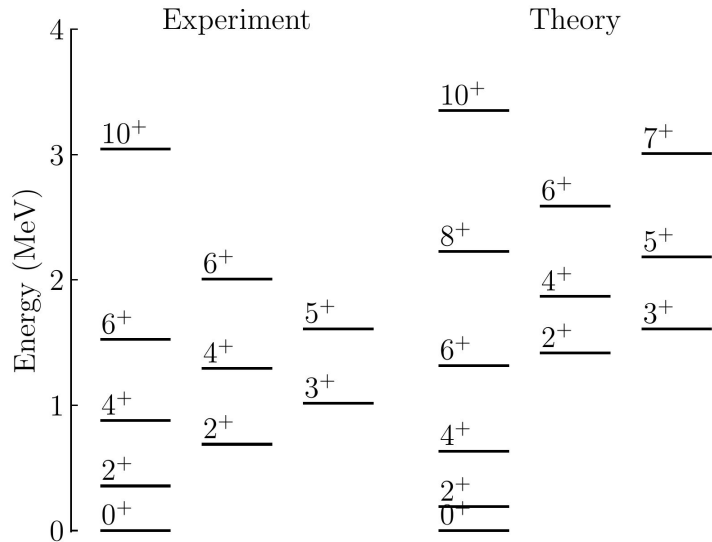
## Triaxial deformation for actinides

- Larger effects with growing  $N$
- reminder:  $\sigma(\text{fission}) < 0.5 \text{ MeV}$
- what other regions does it affect?

# Magnetic moments



# What is the effect on **nuclear level densities**?



## Broken symmetries impact NLDs

- **axial** rotors give rise to **sparse** spectra
- **triaxial** rotors have **dense** spectra
- simple models for collective effects

## Level densities with BSkG2

F. Giacoppo et al., PRC 90, 054330 (2014).

- not always higher level density
- but a **different energy dependence!**
- systematic calculations underway