

Andy Sproles, ORNL

# Microscopic models of nuclear structure for applications

March 26th 2023



wryssens.com

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**May 2013:** a young Belgian arrives in Bordeaux.....

On the use of Symmetries in Mean Field Codes  
and the Development of MOCCa

Wouter Ryssens

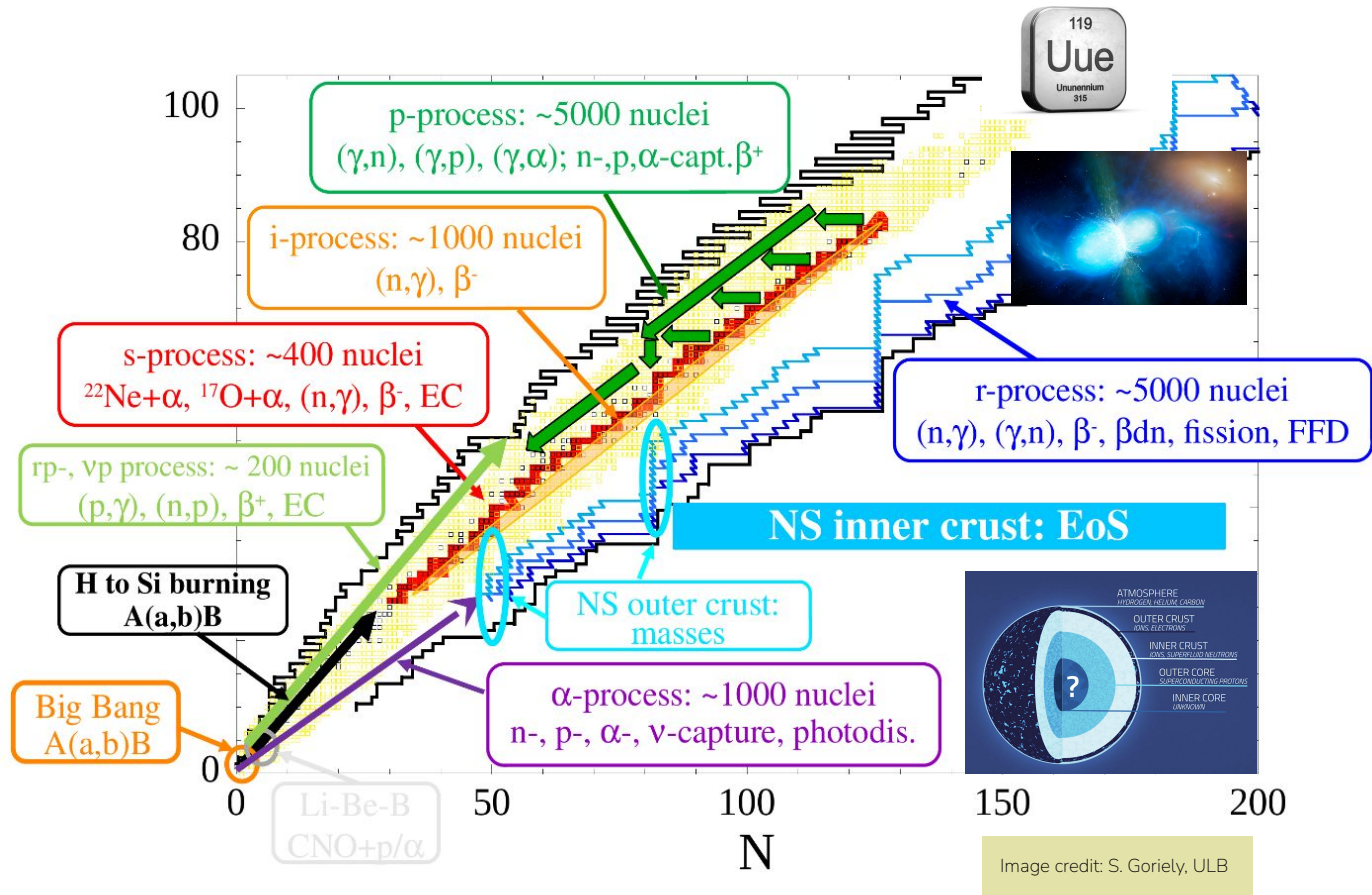
ULB

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2. Theory!
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  - b. deformation
  - c. the BSkG-models
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  - b. rotation, magnetic moments, spins
5. Conclusion, problems and outlook

I. Nuclear theory for applications

# The challenge for nuclear theory: extrapolation!



# The challenge for nuclear theory: extrapolation!

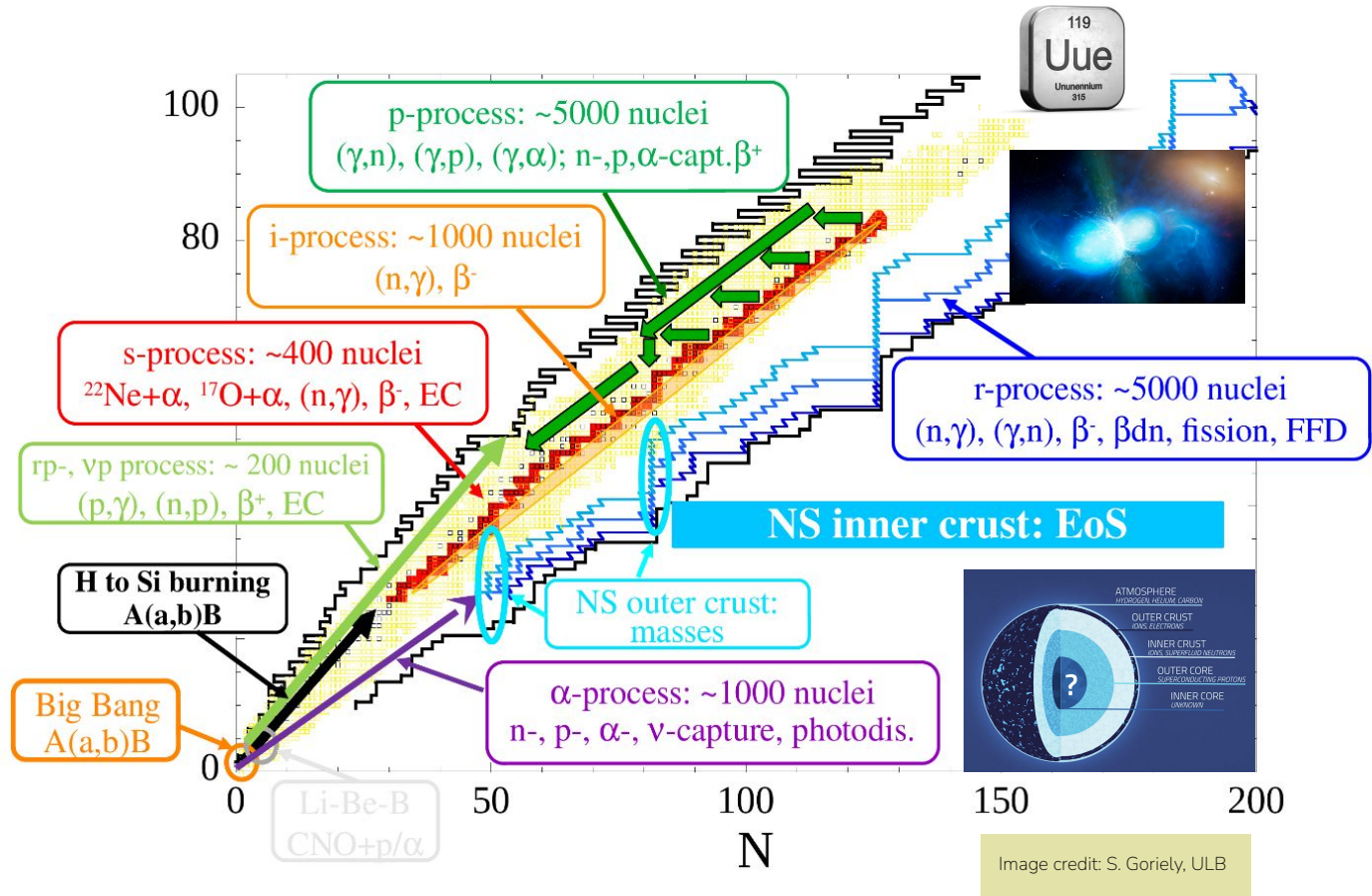
**Extrapolations in**

- nucleon number
- energy
- temperature
- density
- .....

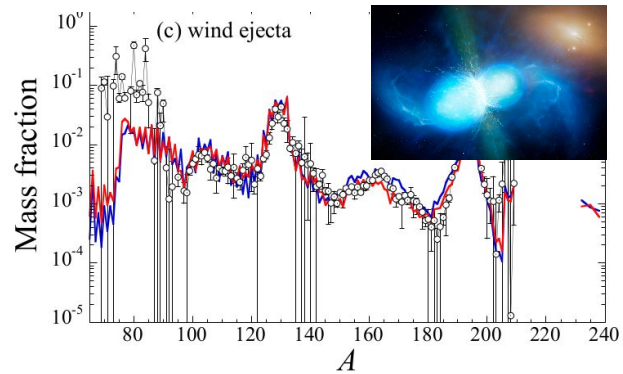
and all of that for

- **~6k** nuclei
- **many** channels

what we need is theory with **predictive power!**

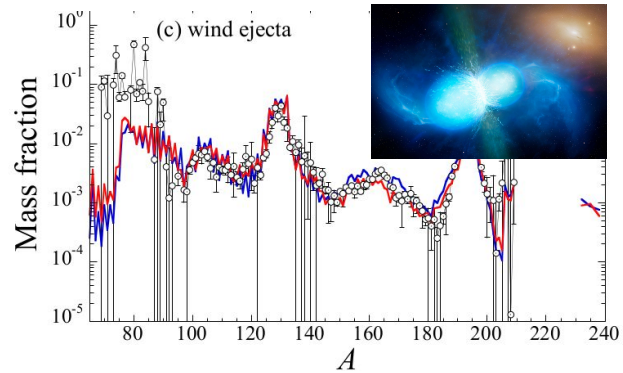


Nucleosynthesis



# Large-scale, microscopic structure models

## Nucleosynthesis





# Large-scale, microscopic structure models

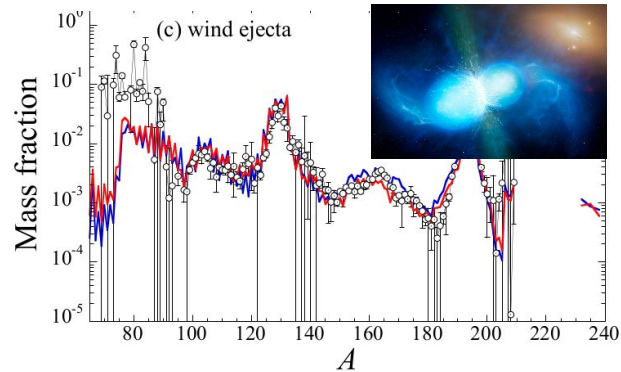
## Structure details

- deformations
- level schemes
- optical potential

## Masses

## Nuclear EOS

## Nucleosynthesis



# Large-scale, microscopic structure models

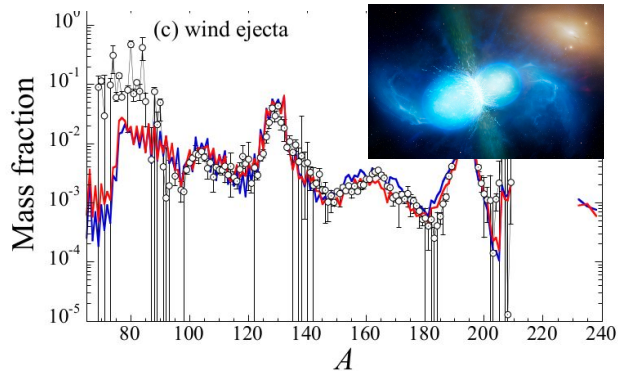
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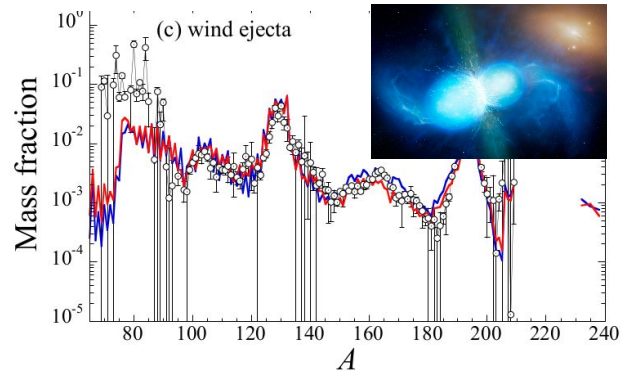
## Masses

## Nucleosynthesis

## Nuclear EOS

## Neutron star structure

## Hydrodynamic simulations



# Large-scale, microscopic structure models

## Structure details

- deformations
- level schemes
- optical potential

## Reaction theory

- reaction rates
- reaction products

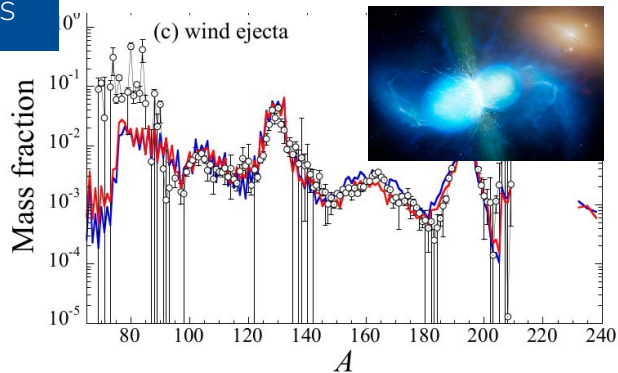
## Masses

## Nucleosynthesis

## Nuclear EOS

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## Hydrodynamic simulations



Low-energy experiments

Large-scale, microscopic structure models

Structure details

- deformations
- level schemes
- optical potential

Masses

Nuclear EOS

Heavy ion collisions

Reaction theory

- reaction rates
- reaction products

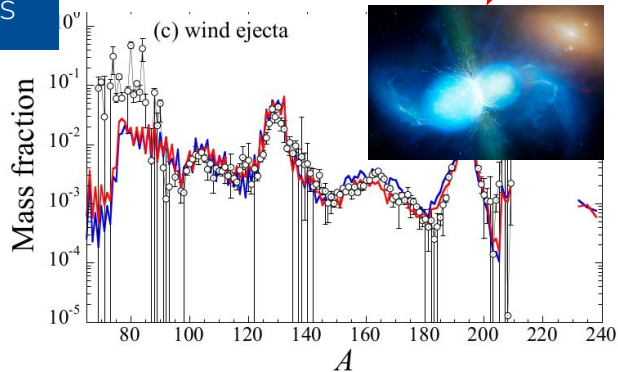
Nucleosynthesis

Observation

Neutron star structure

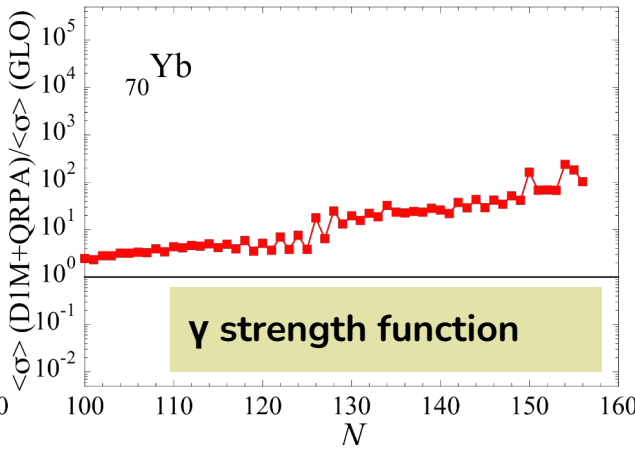
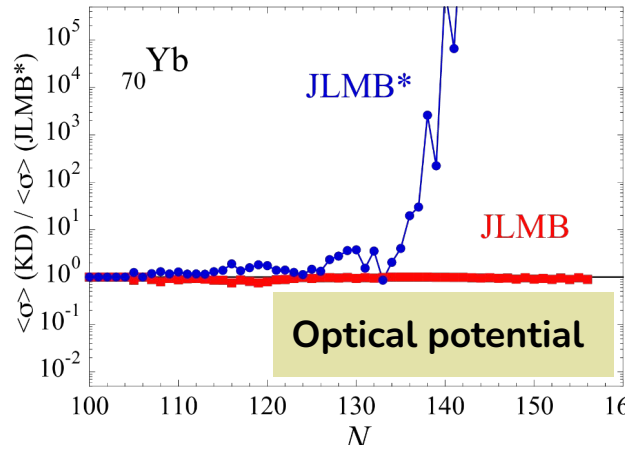
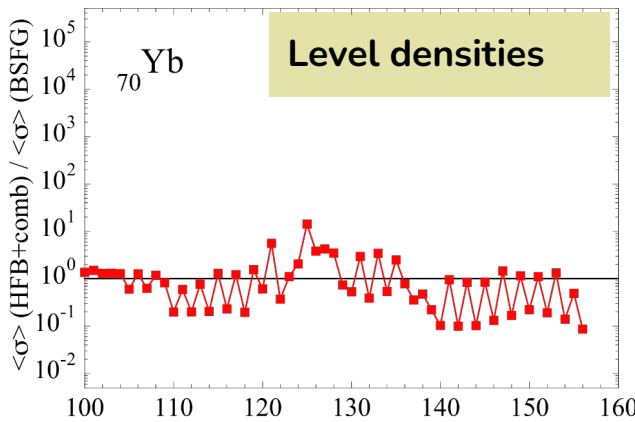
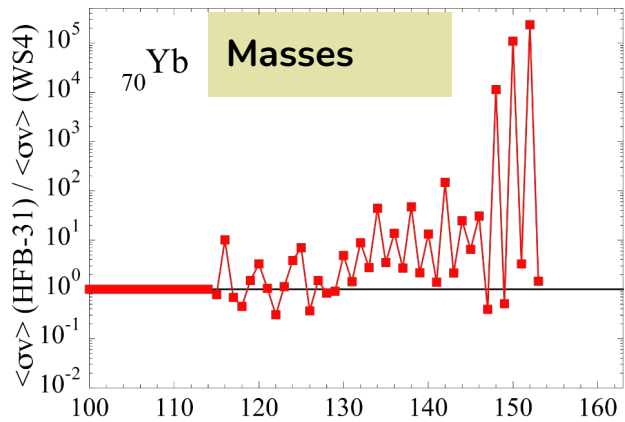
Hydrodynamic simulations

Reaction experiments



# Models can make all the difference...

S. Goriely, EPJA 59, 16 (2023).



## II. EDFs, deformation and the BSkG models

# Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

$$E \sim \int d^3r \left[ C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$





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## Strong points

- starts from **effective interaction**
- **wave function** with individual nucleons
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## How does experimental data enter this?

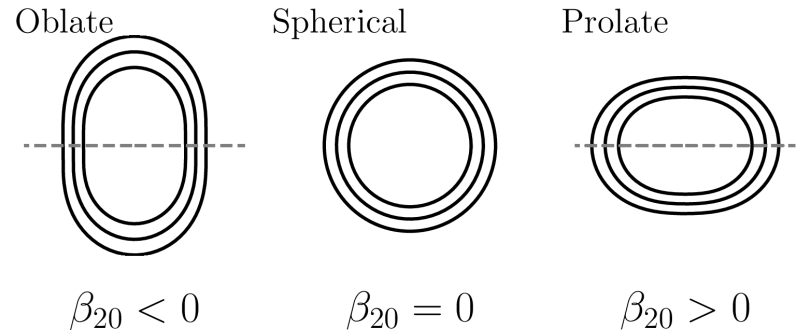
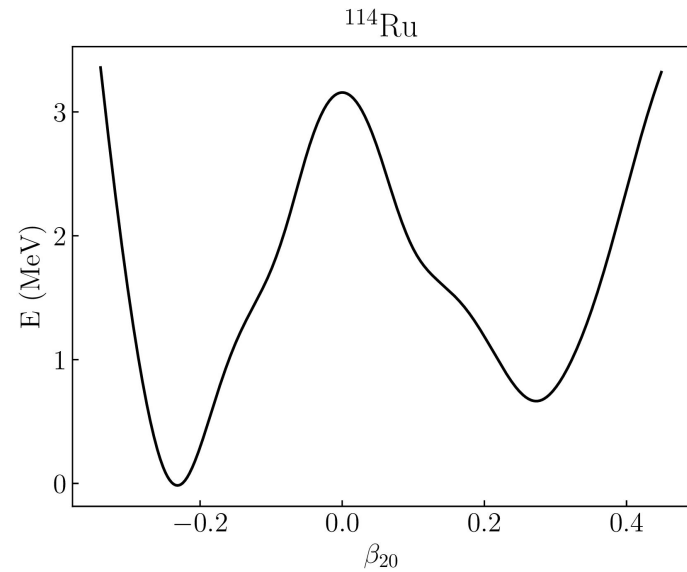
1. Selection and weighting of **fitting data**
2. **Functional form** of the EDF
3. Complexity of the **wave function**

# Nuclear deformation

- the many-body problem is complex
- look for solutions in a given variational space
  - ≈ independent-particle states
  - ≈ Hartree-Fock states
  - ≈ Hartree-Fock-Bogoliubov states
  - ≈ mean-field states
  - ≈ .....

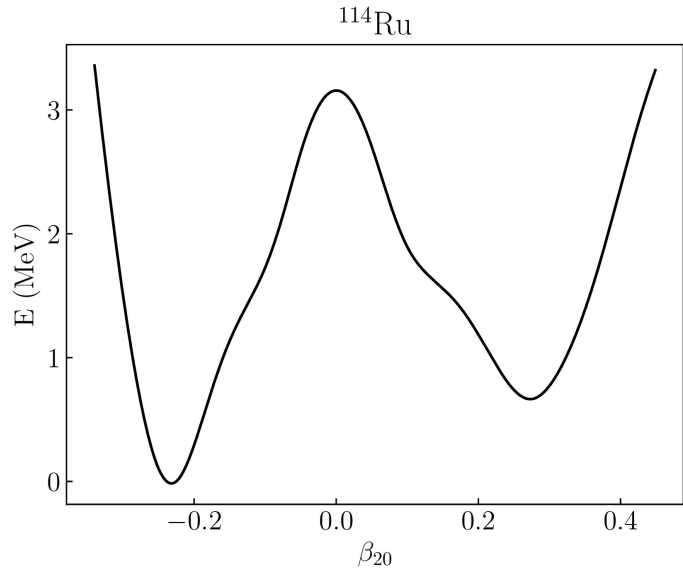
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Symmetry broken

leads to:

Rotational symmetry

Multipole moments

Time-reversal

Angular momentum

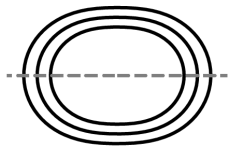
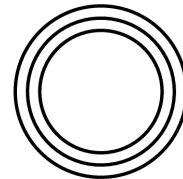
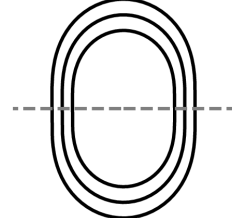
Particle number

Pairing

Oblate

Spherical

Prolate



$\beta_{20} < 0$

$\beta_{20} = 0$

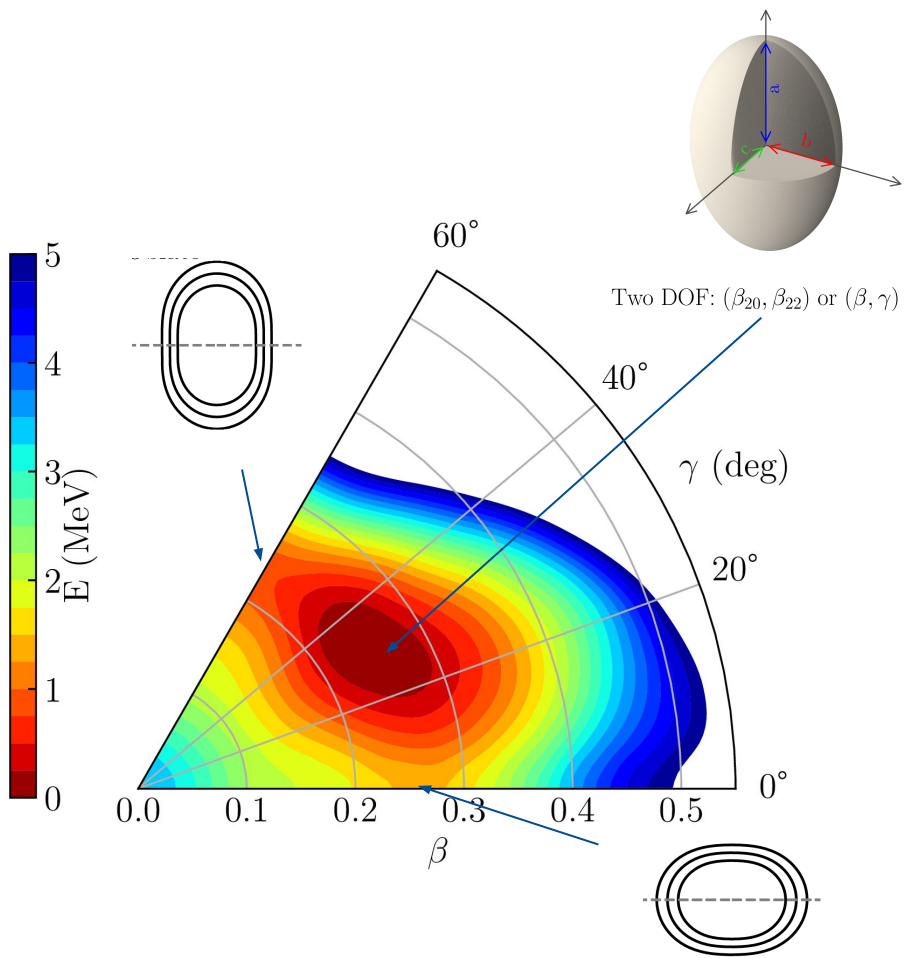
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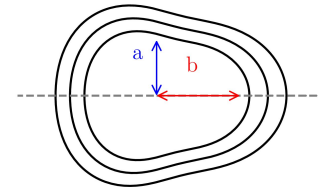
Time-reversal

Angular momentum

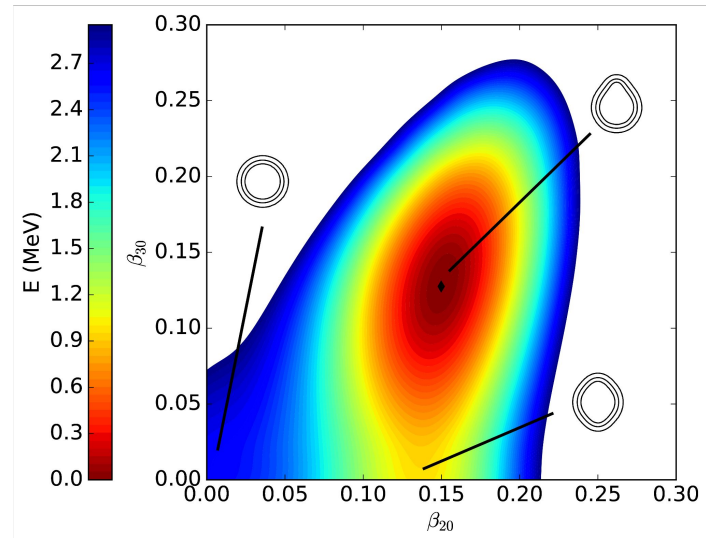
Particle number

Pairing

Reflection-asymmetric (RA)



Two DOF:  $\beta_{20}, \beta_{30}$



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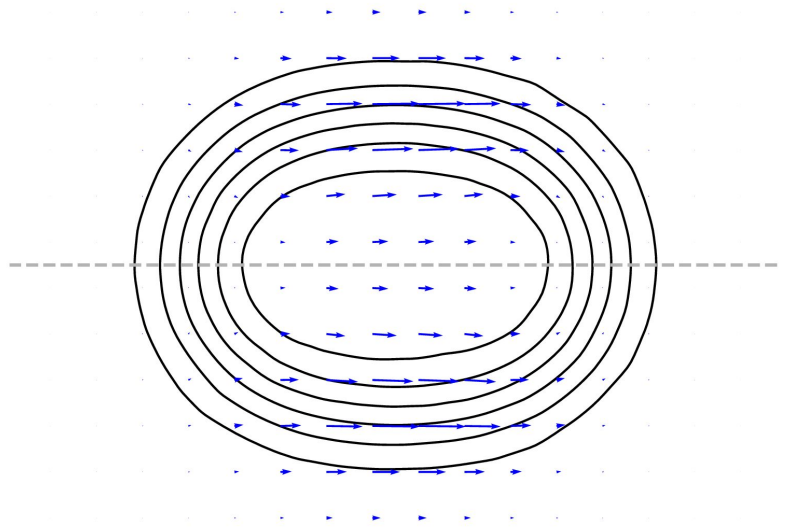
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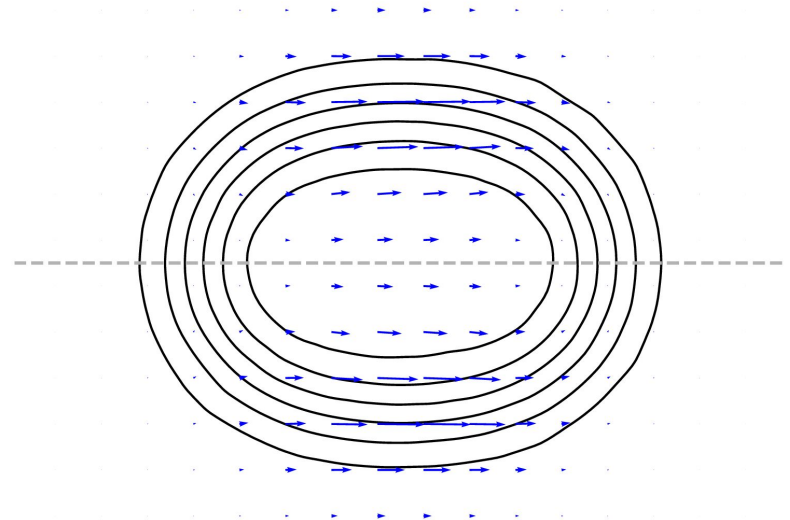
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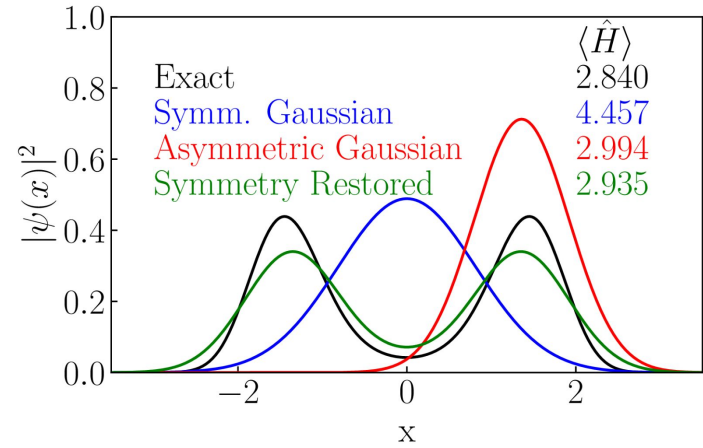
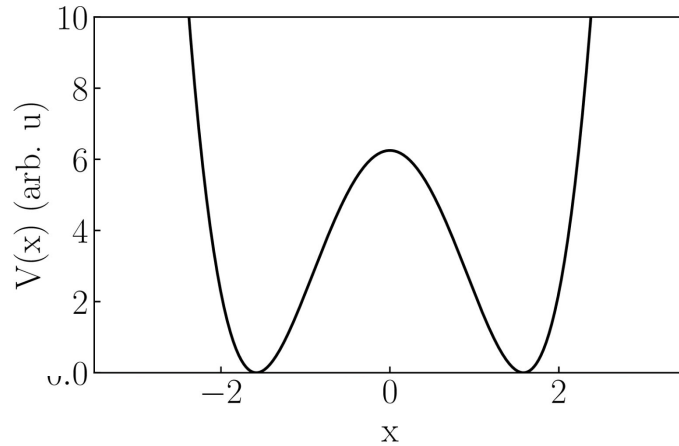
Particle number

Pairing



All of these have experimental signatures!

# Nuclear deformation



## A toy problem

- Gaussians as trial wavefunctions
- Symmetric trial state does badly
- Symmetry-broken solution works!
- ... but misses quantum numbers
- symmetry restoration solves this issue

# The Brussels fit strategy

As much **microscopic** physics as possible:

- simple but symmetry-breaking wave functions
- wave functions determined by eff. interaction
- generalized effective interactions
- all ingredients deduced from wave functions
- all observables deduced from wave functions

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As much **experimental data** as we can muster:

- ground state properties
  - ! 2457 masses of AME20
  - 884 charge radii
  - pairing properties
- fission properties
  - 45 primary & secondary barriers
  - 28 isomer excitation energies
- Infinite nuclear matter properties

# The Brussels fit strategy

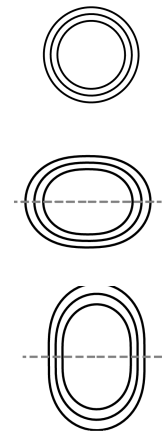
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  - ! **2457** masses of AME20
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Year	Name	Masses	RMS (MeV)
1977		142	0.97
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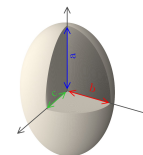
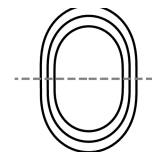
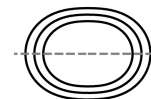
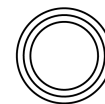
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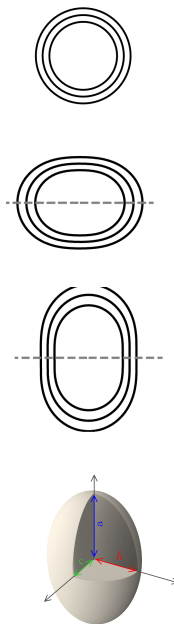
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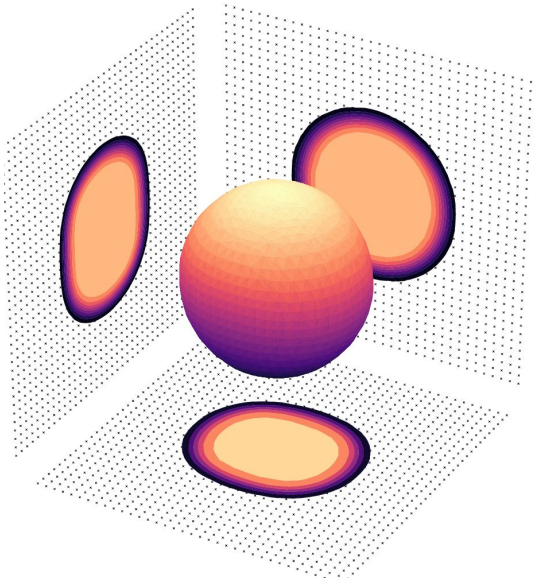
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We focus on the masses,  
but not at the cost of everything else!

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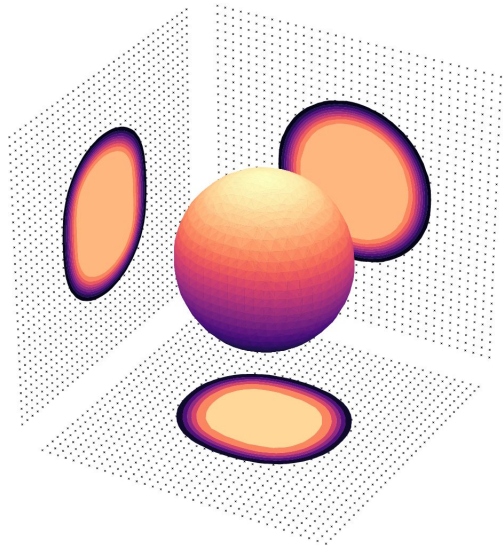
# Brussels-Skyrme-on-a-Grid: tools



## 3D coordinate space

- shape-agnostic
- numerical accuracy

# Brussels-Skyrme-on-a-Grid: tools



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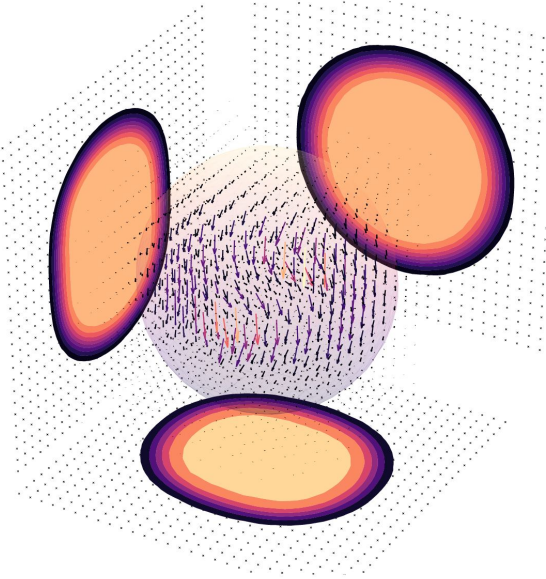
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## MOCCa

W. R. PhD Thesis, ULB (2016).

- **efficient and robust**
- symmetry choices à volonté

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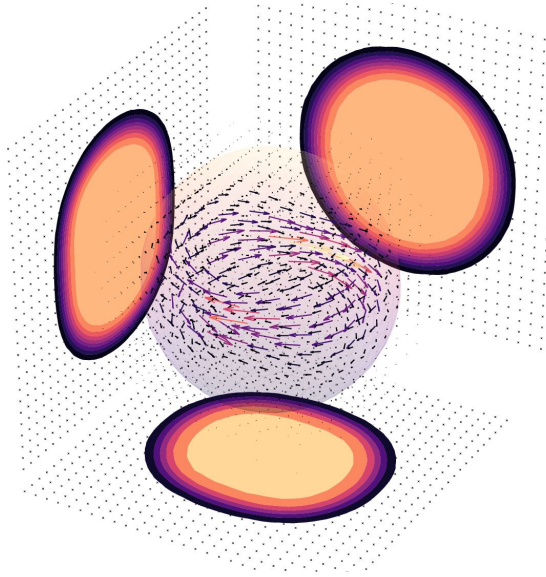
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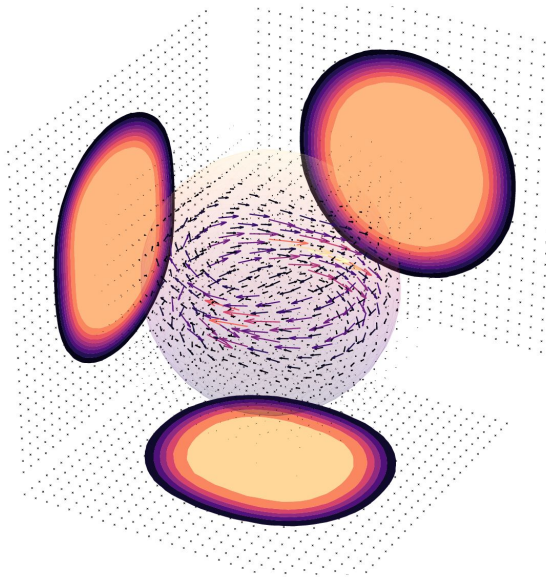
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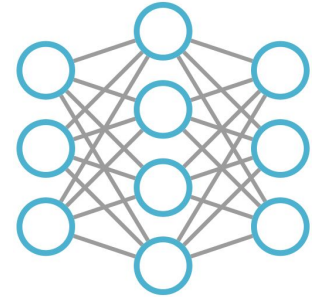
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W. R. PhD Thesis, ULB (2016).

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## ML

G. Scamps et al., EPJA 57, 333 (2021).

- emulate MOCCa
- parameter space exploration
- **No actual predictions!**

# The BSkG models: the raw numbers

## BSkG1

- full 3D representation

## BSkG2

- full 3D representation
- time-reversal symmetry breaking
- fission included in the fit

$\sigma$ (MeV)	BSkG1	BSkG2	HFB-14	FRLDM
<b>2457 masses</b>	<b>0.741</b>	<b>0.678</b>		
884 charge radii (fm)	0.027	0.027		
45 primary barriers	0.87	0.44		
45 secondary barriers	0.86	0.47		
28 Isomer energies	0.45	0.49		

BSkG2: W. Ryssens et al., EPJA 58, 246 (2022).

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).



# The BSkG models: the raw numbers

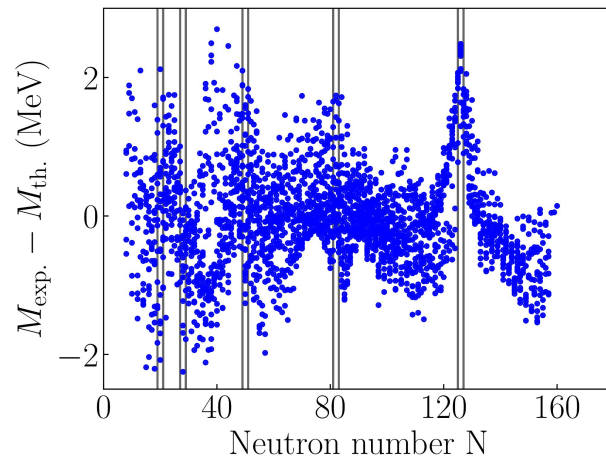
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45 secondary barriers	0.86	0.47		
28 Isomer energies	0.45	0.49		



# The BSkG models: the raw numbers

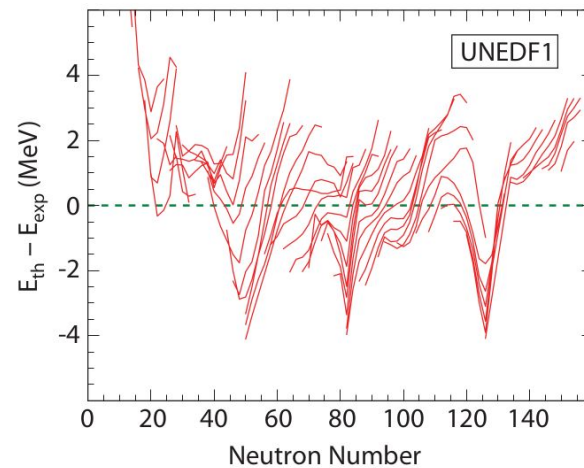
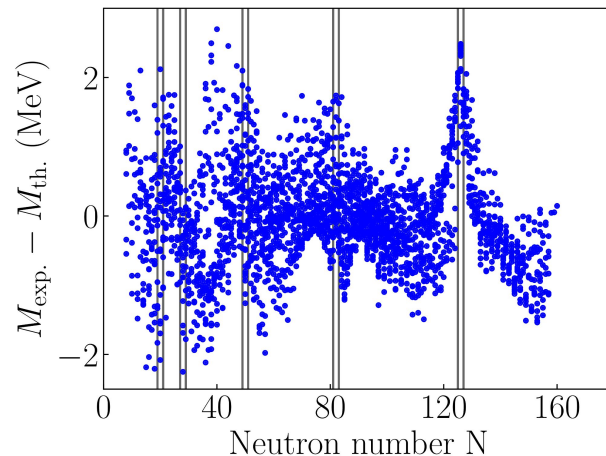
## BSkG1

- full 3D representation

## BSkG2

- full 3D representation
- time-reversal symmetry breaking
- fission included in the fit

$\sigma$ (MeV)	BSkG1	BSkG2	HFB-14	FRLDM
<b>2457 masses</b>	<b>0.741</b>	<b>0.678</b>		
884 charge radii (fm)	0.027	0.027		
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BSkG2: W. Ryssens et al., EPJA 58, 246 (2022).

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).

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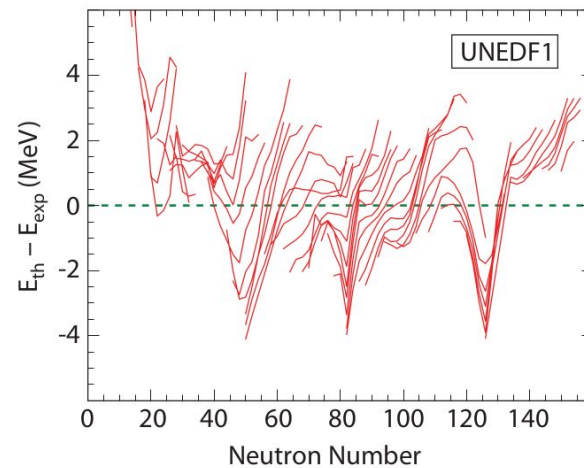
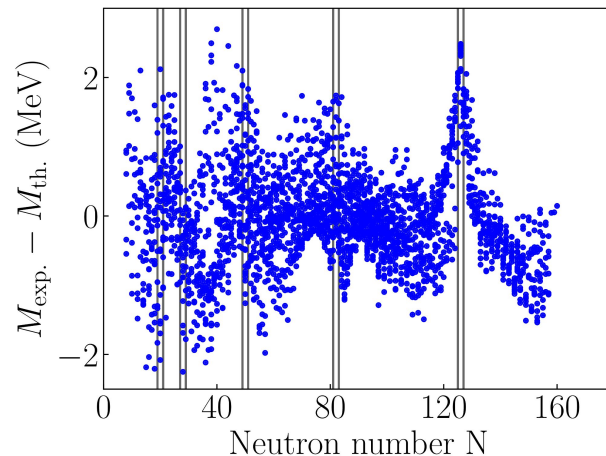
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$\sigma$ (MeV)	BSkG1	BSkG2	HFB-14	FRLDM
<b>2457 masses</b>	<b>0.741</b>	<b>0.678</b>	<b>0.729</b>	<b>0.808</b>
884 charge radii (fm)	0.027	0.027	0.039	-
45 primary barriers	0.87	0.44	0.61	0.79
45 secondary barriers	0.86	0.47	0.70	1.35
28 Isomer energies	0.45	0.49	0.93	1.04



BSkG2: W. Ryssens et al., EPJA 58, 246 (2022).

BSkG1: G. Scamps et al., EPJA 57, 333 (2021).

III. Why do we do these complex things?

# Interlude: why do we do these complex things?

## Mic-mac approaches?



competitive in rms



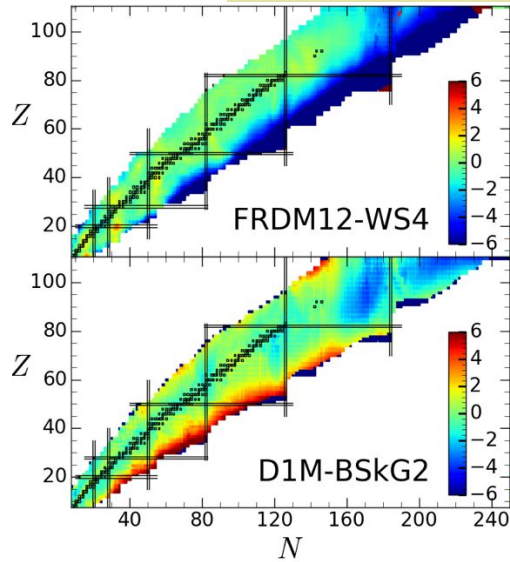
multiple observables

## Machine learning?

## Ab Initio?

# Interlude: why do we do these complex things?

S. Goriely, EPJA 59, 16 (2023).



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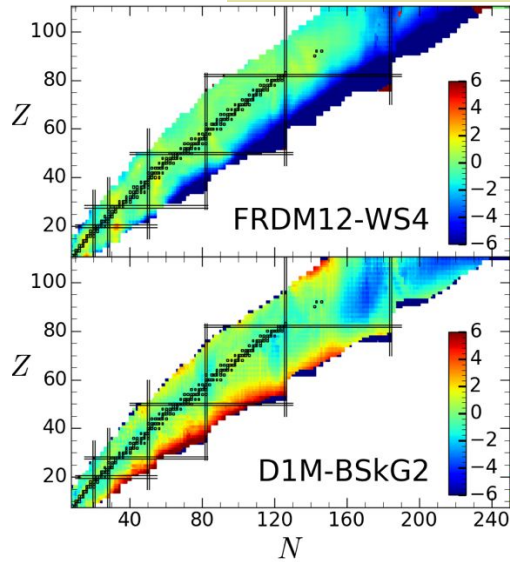
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- ✓ multiple observables
- ✗ comparatively unstable
- ✗ no link mic.  $\leftrightarrow$  mac.

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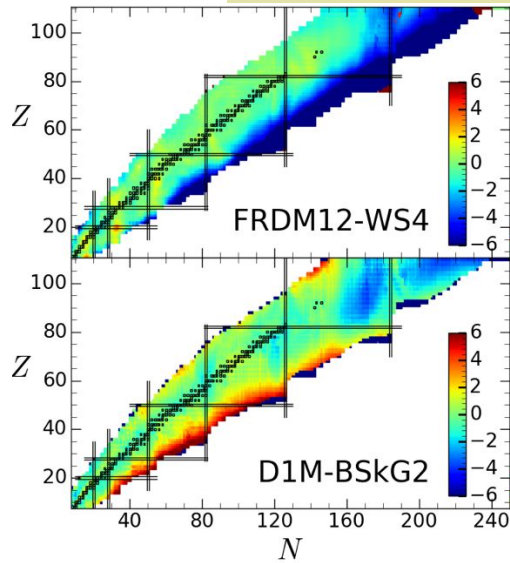
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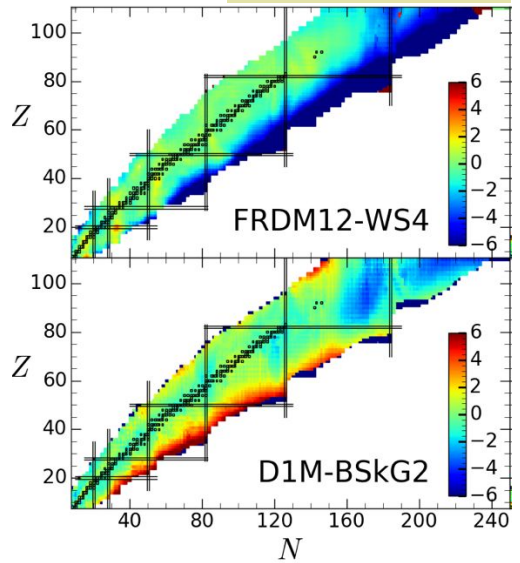
- ✓ absolute champion in rms
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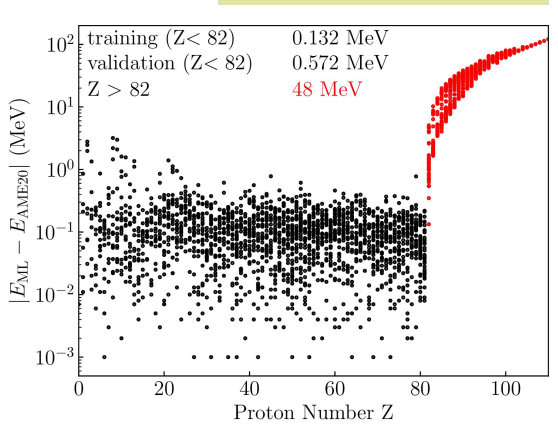


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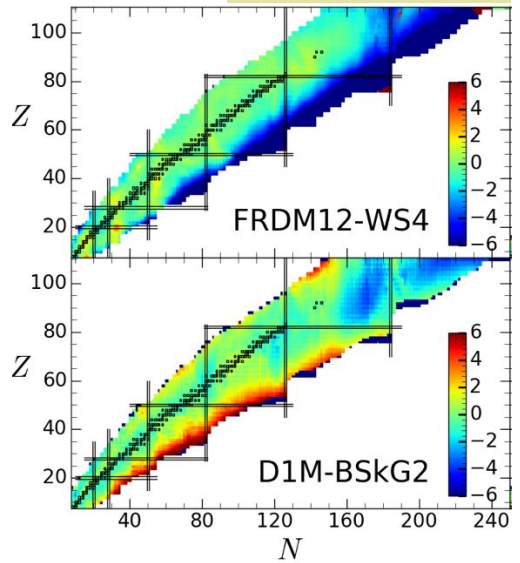
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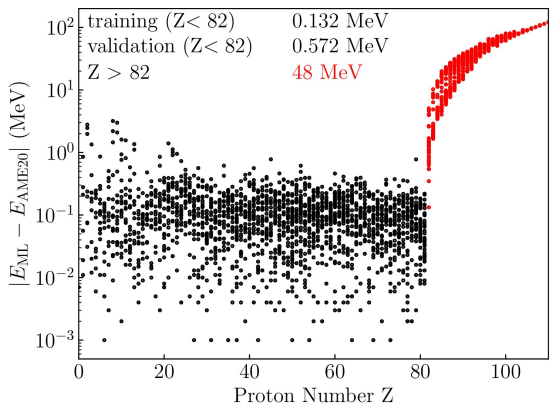
-

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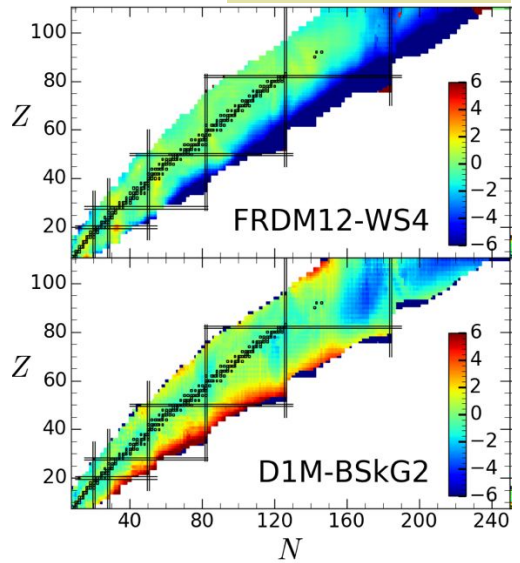
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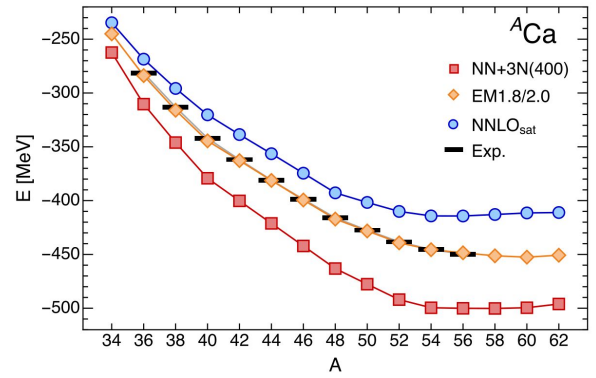
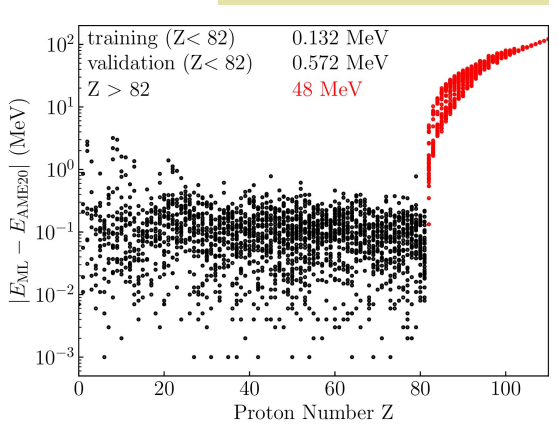
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- ✓ “truly” microscopic
- ✓ multiple observables

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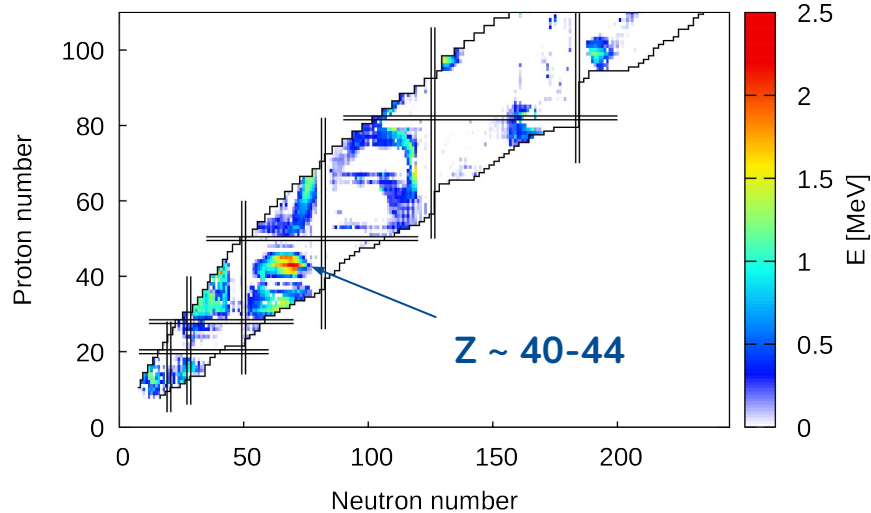
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## Ab Initio?

- ✓ error quantification
- ✓ “truly” microscopic
- ✓ multiple observables
- ✗ infeasible at scale (for now)
- ✗ not competitive on rms (for now)

# IV. Observables

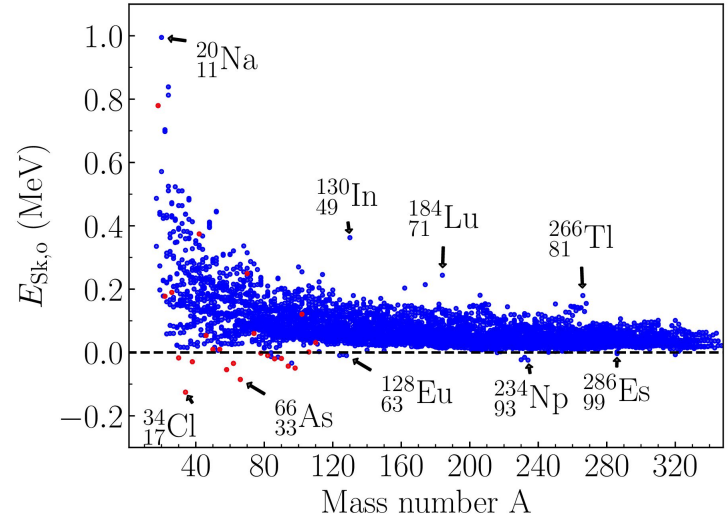
# Observables: masses



## Triaxial deformation

G. Scamps et al., EPJA 57, 333 (2021).

- most nuclei remain axial
- several regions of triaxiality
- large effects from 0.5-2 MeV.



## Time-reversal symmetry breaking

W.R. et al., EPJA 58, 246 (2022).

- reduces binding energy
- but small contribution...
- ... and grows smaller with system size

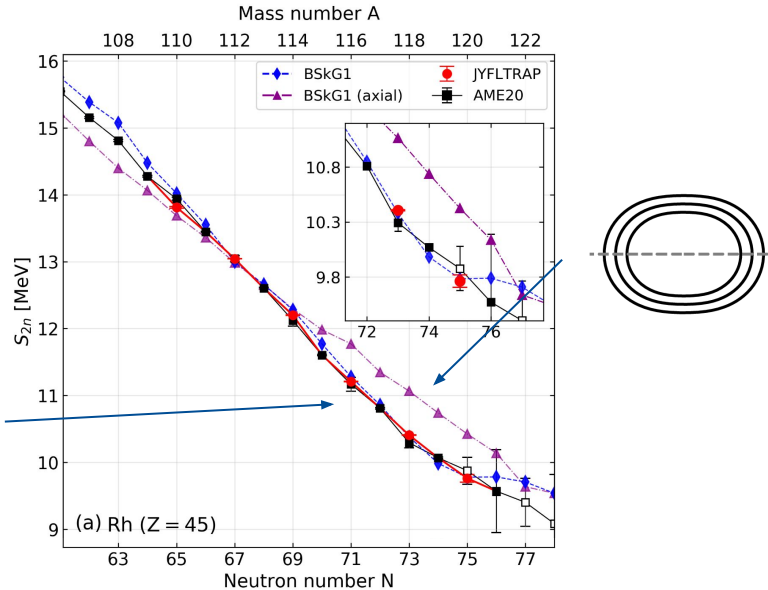
# Observables: masses

## Rh isotopes, nice!

M. Hukkanen et al., PRC 107, 014306 (2023).

- masses of  $^{110,112,114,116,118}\text{Rh}$  g.s. measured
- absolute masses within  $\sim 700$  keV

# Observables: masses



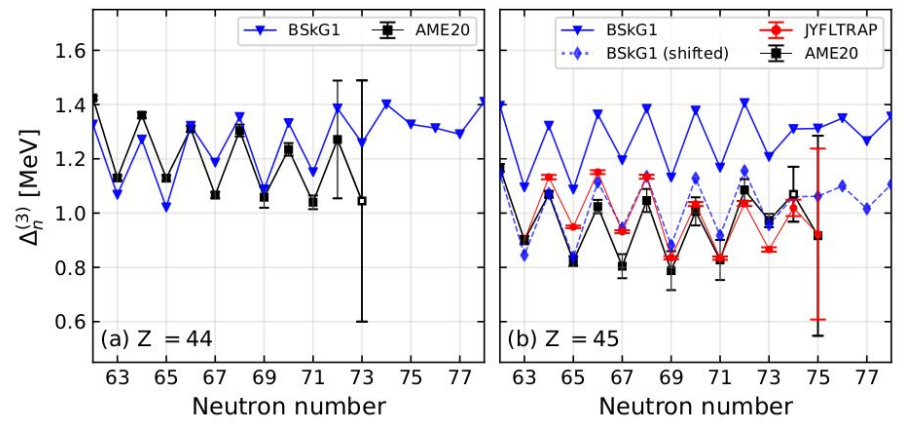
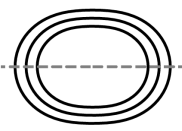
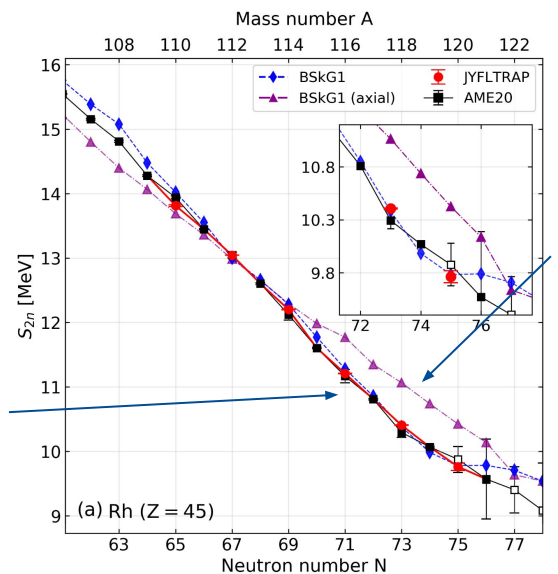
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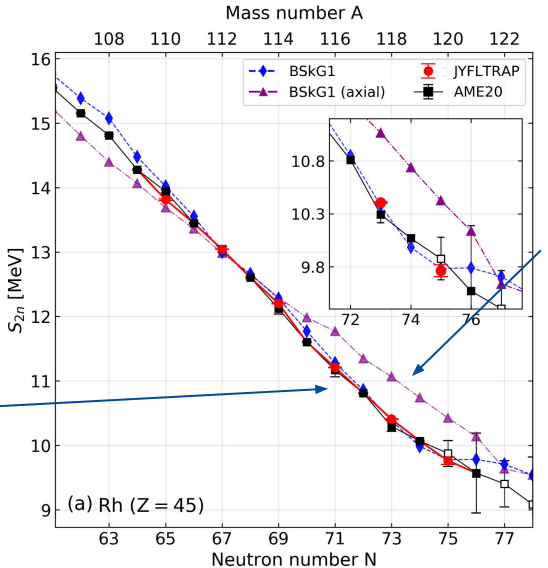
## Rh isotopes, not-so-nice!

W.R. et al., arXiv:2211.03667 [nucl-th].

- total failure to reproduce  $\Delta_n^{(3)}$  ....
- ... but works in neighbouring chains!



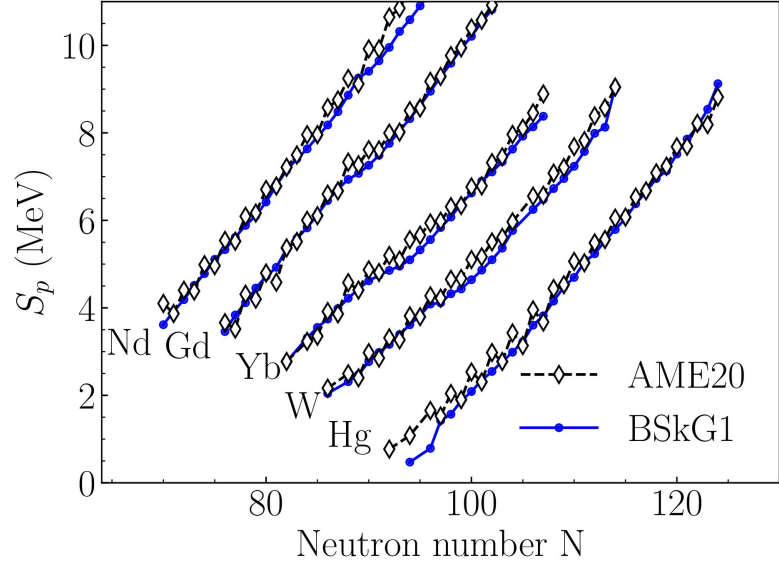
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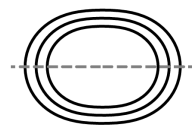
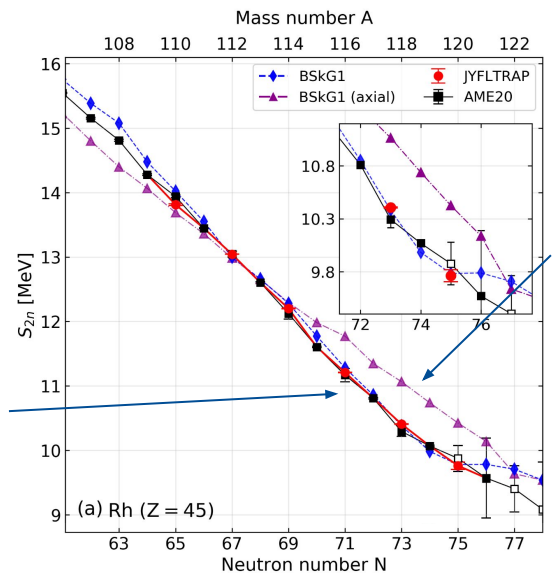


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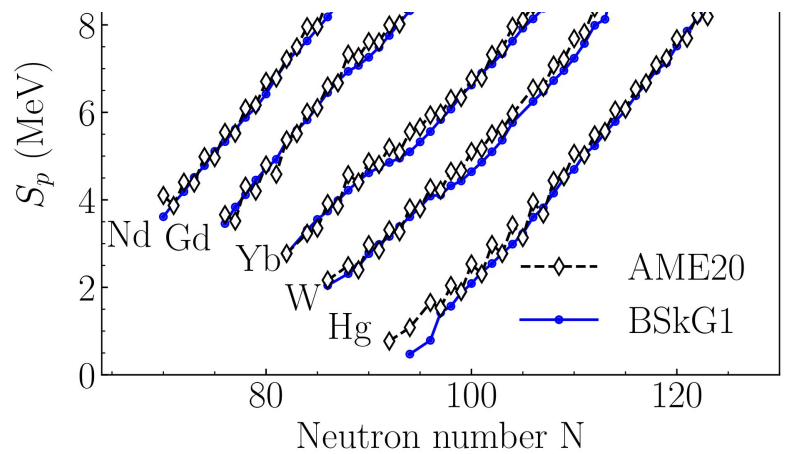
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- is linked to interaction odd-n and odd-p?

# Observables: masses



$$E \sim \int \mathbf{j}_p(\mathbf{r}) \cdot \mathbf{j}_n(\mathbf{r}) + \mathbf{s}_p(\mathbf{r}) \cdot \mathbf{s}_n(\mathbf{r})$$



## Rh isotopes, nice!

M. Hukkanen et al., PRC 107, 014306(2023).

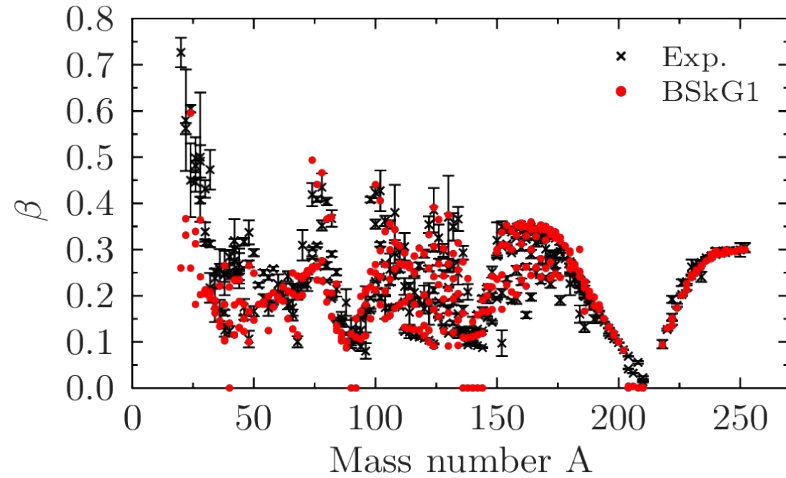
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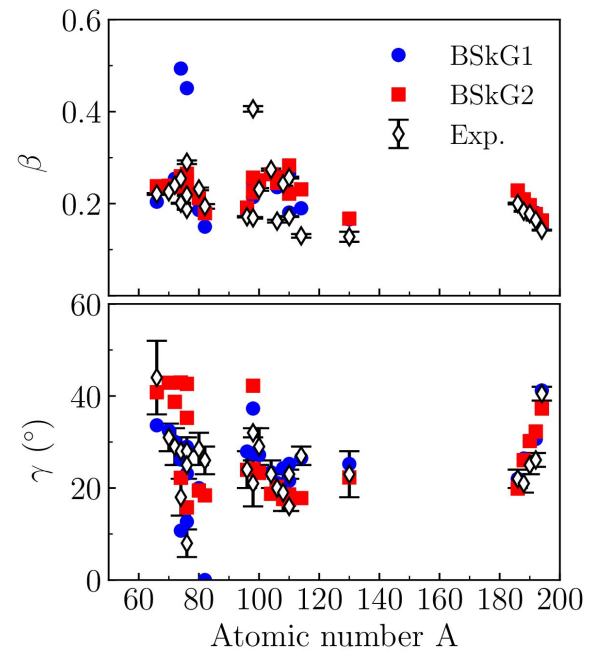
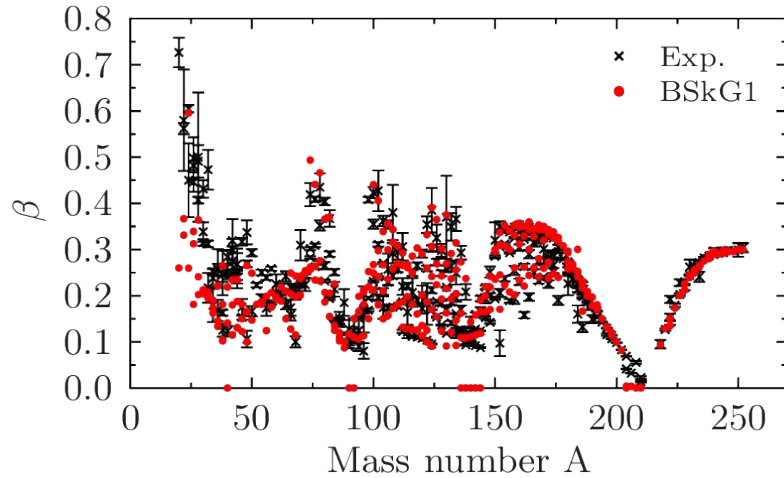


## Global systematics

S. Raman, At. Data **78**(1), 1-128 (2001).

- deduced from  $B(E2)$  transitions
- good global reproduction
- particularly for heavy nuclei

# Observables: deformations



## Global systematics

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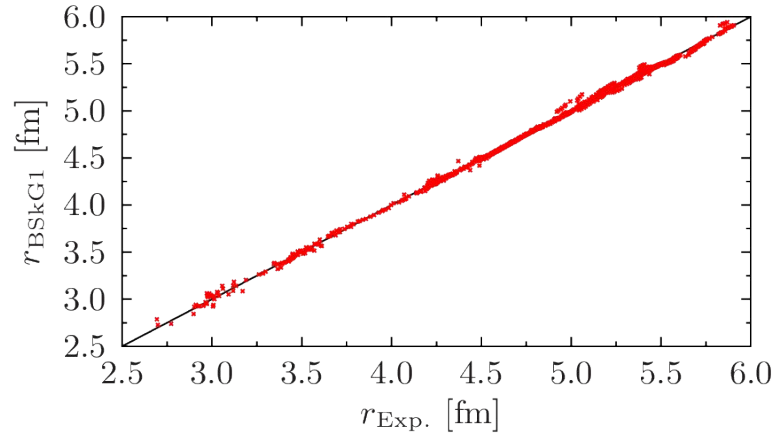
- deduced from  $B(E2)$  transitions
- good global reproduction
- particularly for heavy nuclei

## Triaxial deformation?

Thanks to M. Zielinska!

- ALL available COULEX data
- very good agreement (within model limitations)

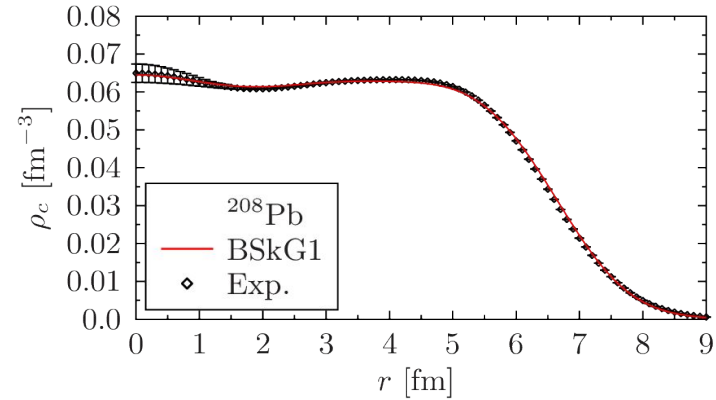
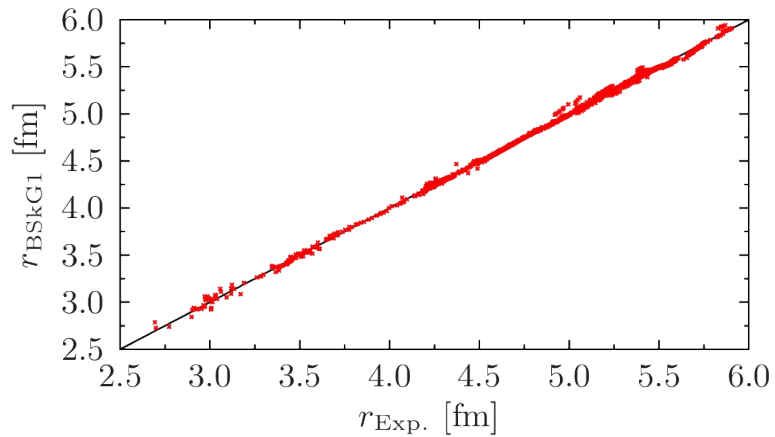
Observables: charge densities/radii



## Systematics and details of charge densities

✓ rms (charge radii)  $\sim 0.027$  fm

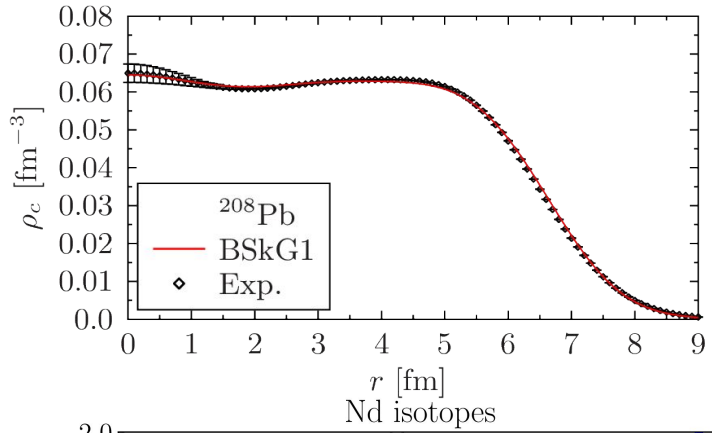
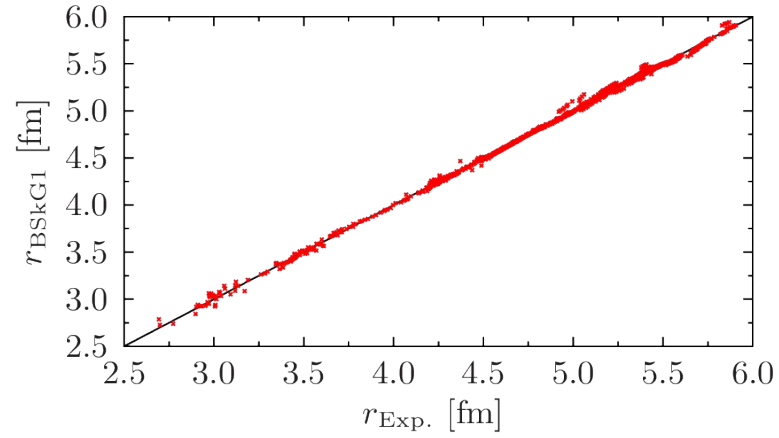
# Observables: charge densities/radii



## Systematics and details of charge densities

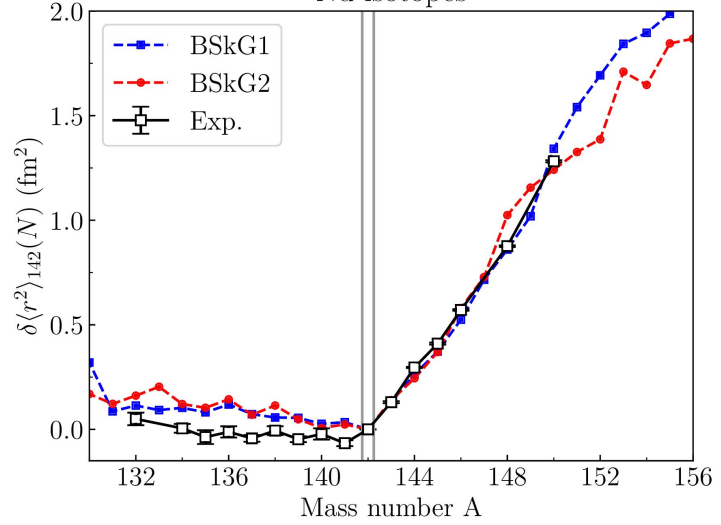
- ✓ rms (charge radii)  $\sim 0.027$  fm
- ✓ complete charge densities

# Observables: charge densities/radii

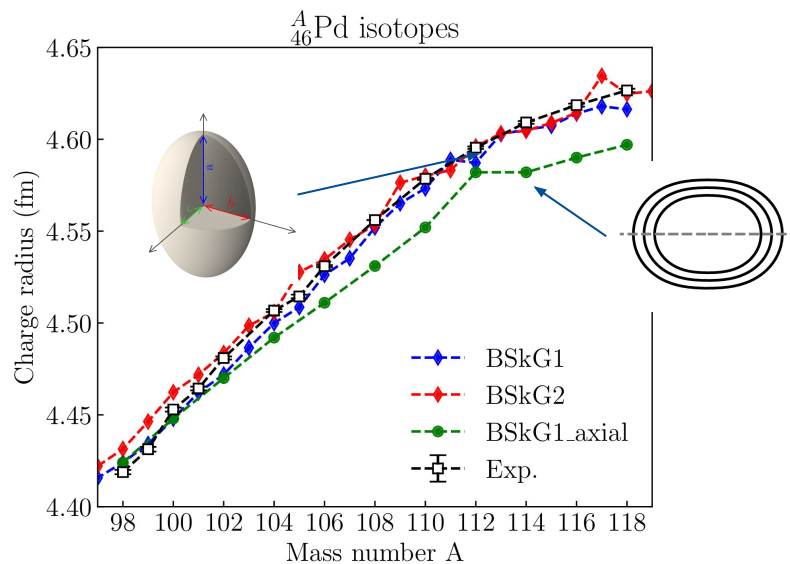


## Systematics and details of charge densities

- ✓ rms (charge radii)  $\sim 0.027$  fm
- ✓ complete charge densities
- ✓ dramatic evolution with particle number  
..... linked to deformation!



# Observables: charge densities/radii



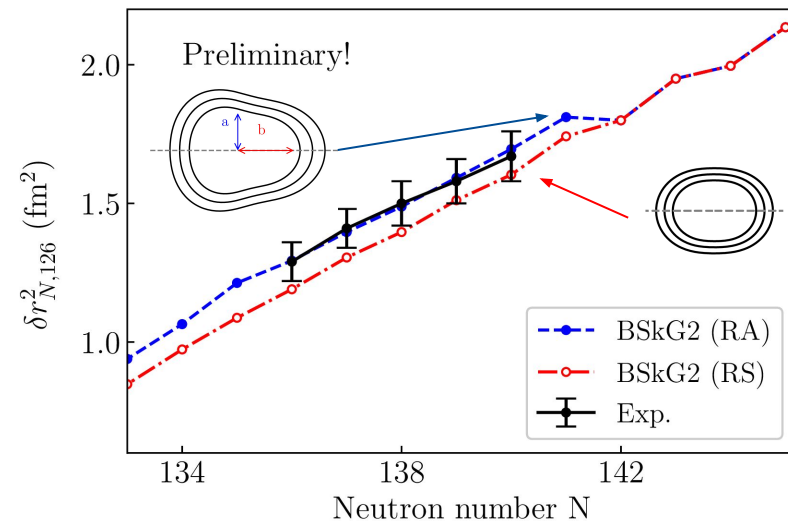
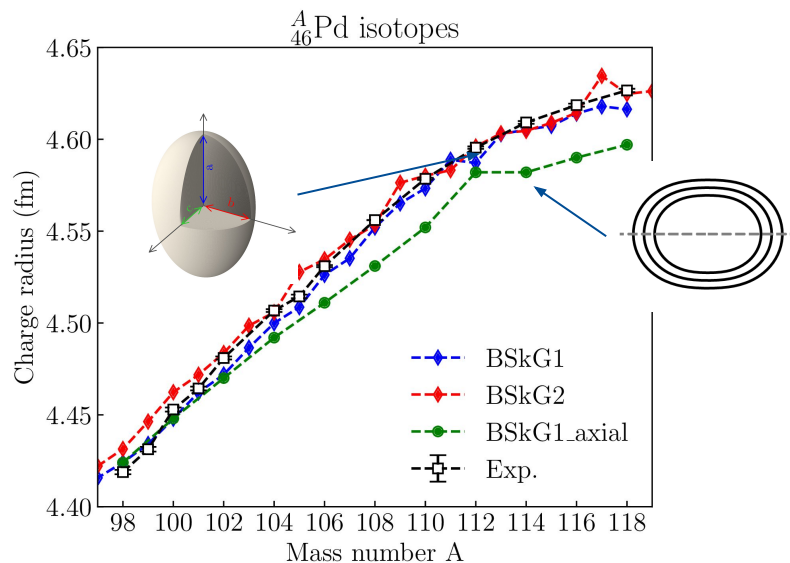
## Pd radii

S. Geldhof, PRL 128, 152501 (2022).

- radii are sensitive to ALL deformation
- not just  $\beta_{20}$ , but also  $\beta_{22}$ !



# Observables: charge densities/radii



## Pd radii

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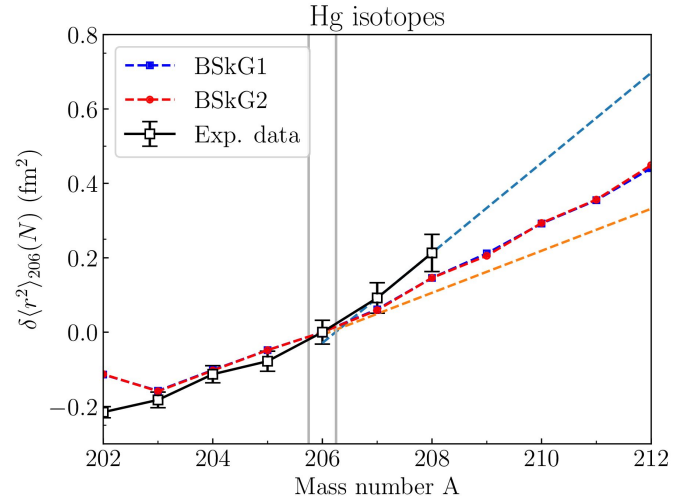
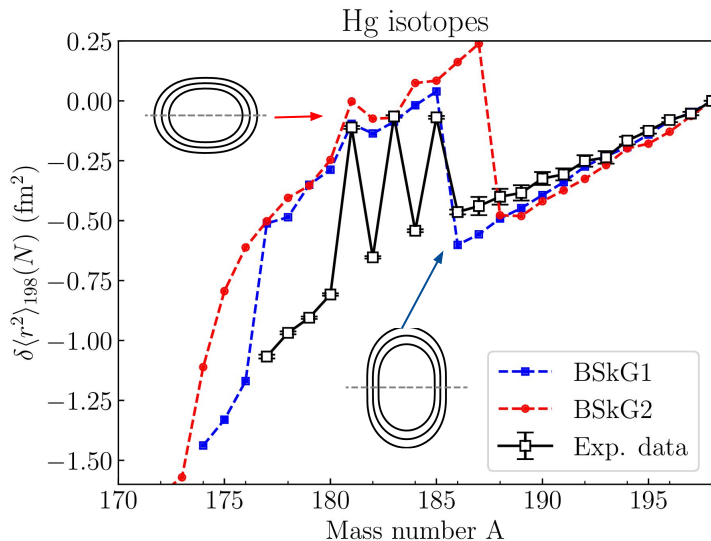
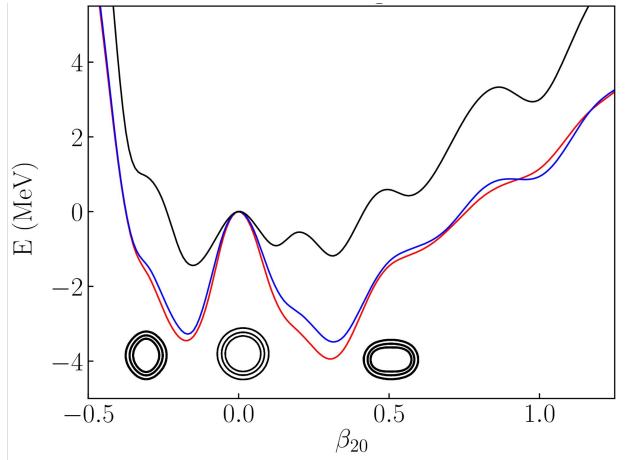
- radii are sensitive to ALL deformation
- not just  $\beta_{20}$ , but also  $\beta_{22}$ !

## Ac radii

E. Verstraelen et al., PRC 100, 044321 (2019).

- .... and also  $\beta_{30}$ !
- predicted end of octupole region
- New experiment ongoing....

# Observables: charge densities/radii



## Hg isotopes are hard!

S. Sels et al., Phys. Rev. C 99 (2019).

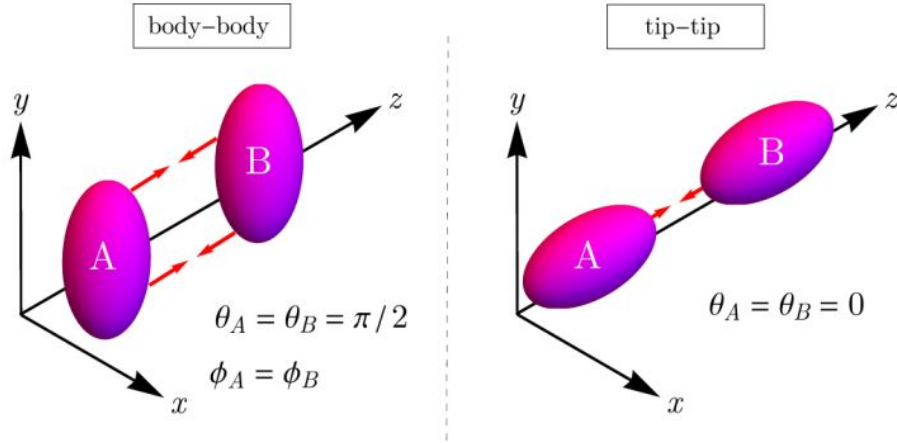
### Staggering:

- rapid shape change
- **multiple, competing** minima

### Kink at N=126

- no obvious interpretation
- also exists at N=20,50,82!
- my bet: single-particle structure

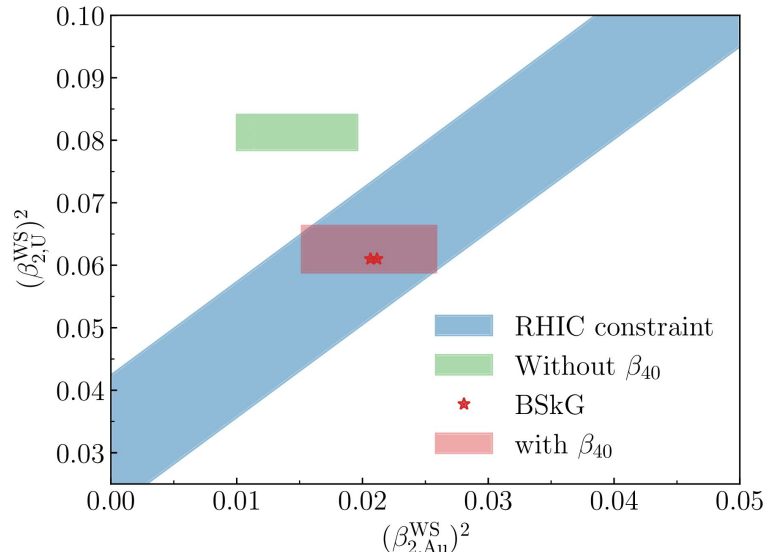
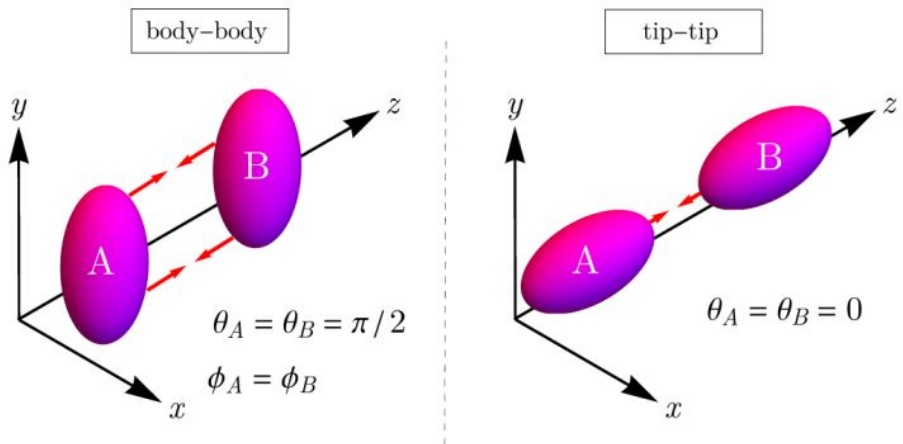
# Observables: densities



## Heavy ion collisions

- shape of the quark-gluon plasma depends on nuclear density
- deformation imprints on the particles detected!

# Observables: densities



## Heavy ion collisions

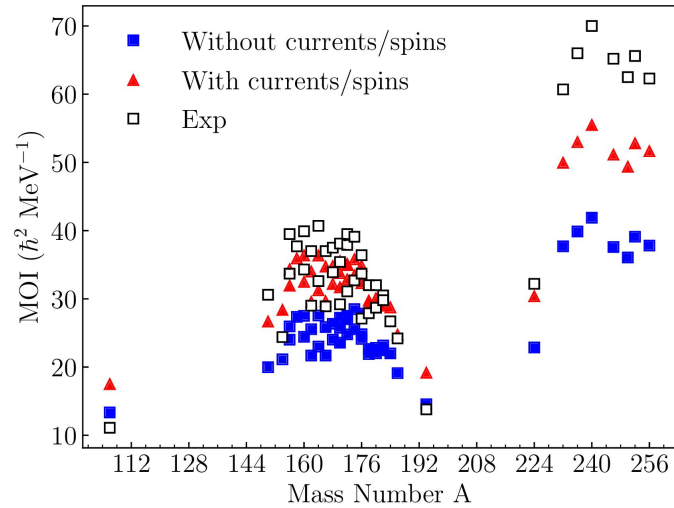
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## Exciting times!

W. R. et al., [arXiv:2302.13617](https://arxiv.org/abs/2302.13617) (2023).

- a priori sensitive to ALL aspects
  - exotic multipole moments
  - matter (!) radii
- Au+Au collisions and U+U collisions can only be described by including  $\beta_{40}$

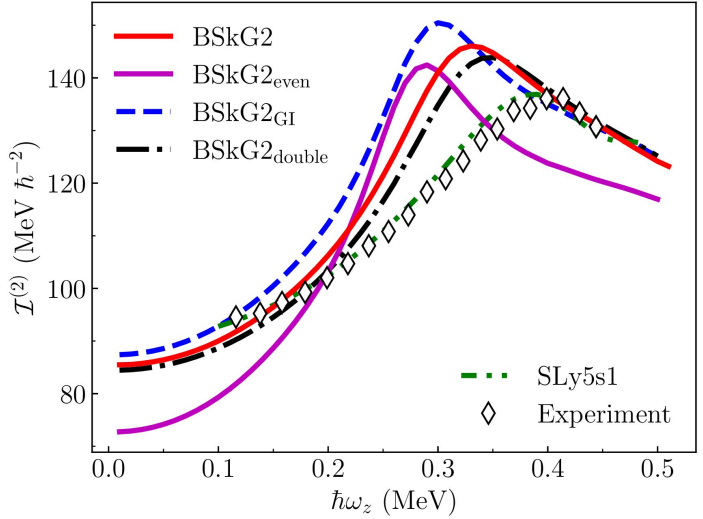
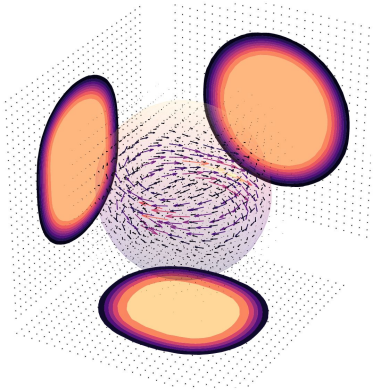
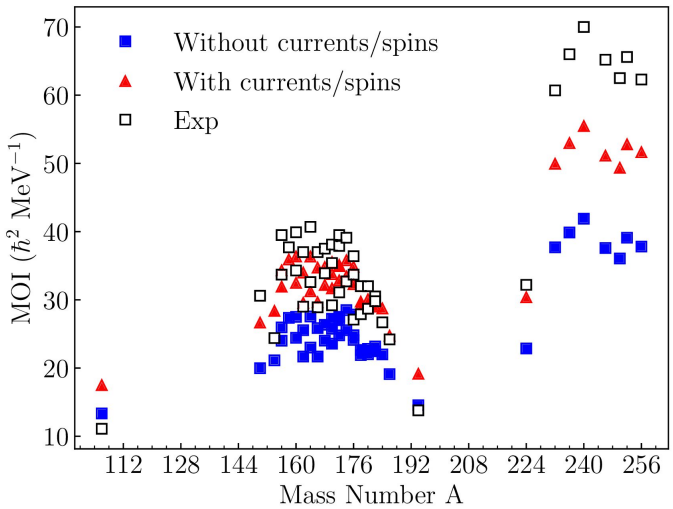
# Observables: rotational properties



## Rotational response of nuclei

- currents and spins contribute to MOI
- ✓ medium-heavy nuclei
- ✗ actinides

# Observables: rotational properties



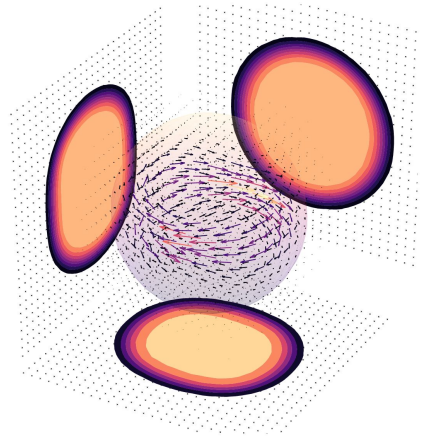
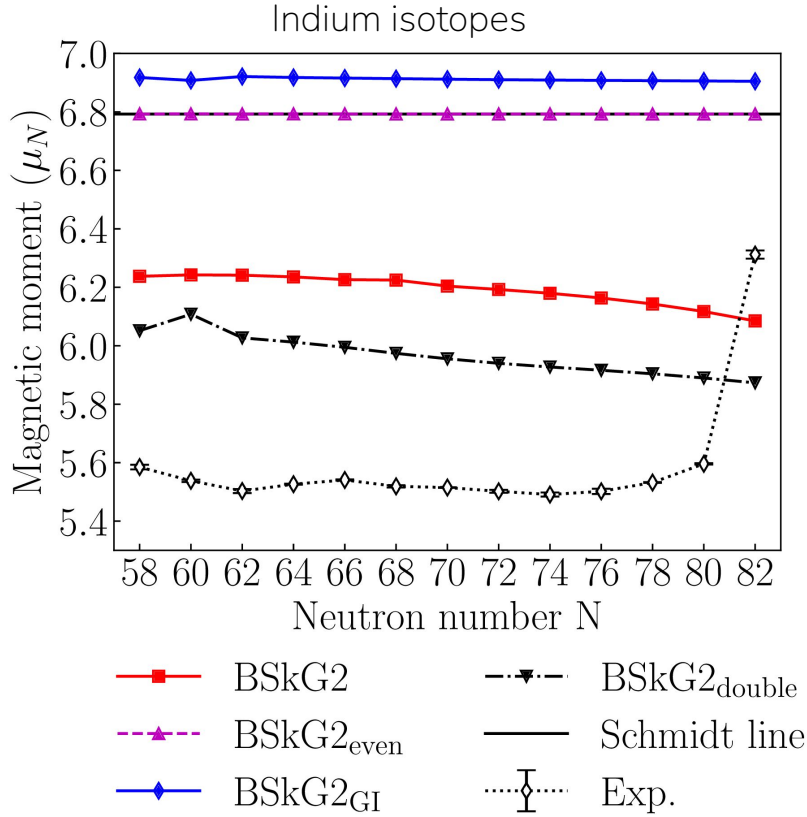
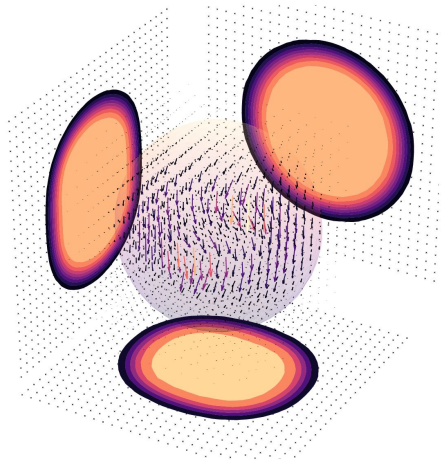
## Rotational response of nuclei

- currents and spins contribute to MOI
- ✓ medium-heavy nuclei
- ✗ actinides

## MOI evolve along rotational bands

- top of the peak = “alignment”
- curve sensitive to currents and spins!

# Observables: magnetic moments



# Observables: g.s. spins and parities

Correct spin-parity?	odd-Z	odd-N	odd-odd
Spherical	82%	68%	33%
Transitional	70%	63%	13%
Axial deformation	25%	40%	19%
Triaxial deformation	28%	27%	15%

## Perhaps the most basic observable....

- g.s. spin and parities require symmetry restoration
- something “quick-n-dirty”
- $J^\pi$  enters reaction rate calculations ....

## ..... but this is “as good” as other models

- all other EDF approaches
- mic-mac approaches
- See:

L. Bonneau et al., PRC 76, 024320 (2007)



## V. Conclusion, problems and outlook

# Conclusion

We build large-scale, microscopic models of nuclear structure for applications.

Microscopic = with **simple** wave functions yet complex **symmetry breaking**.

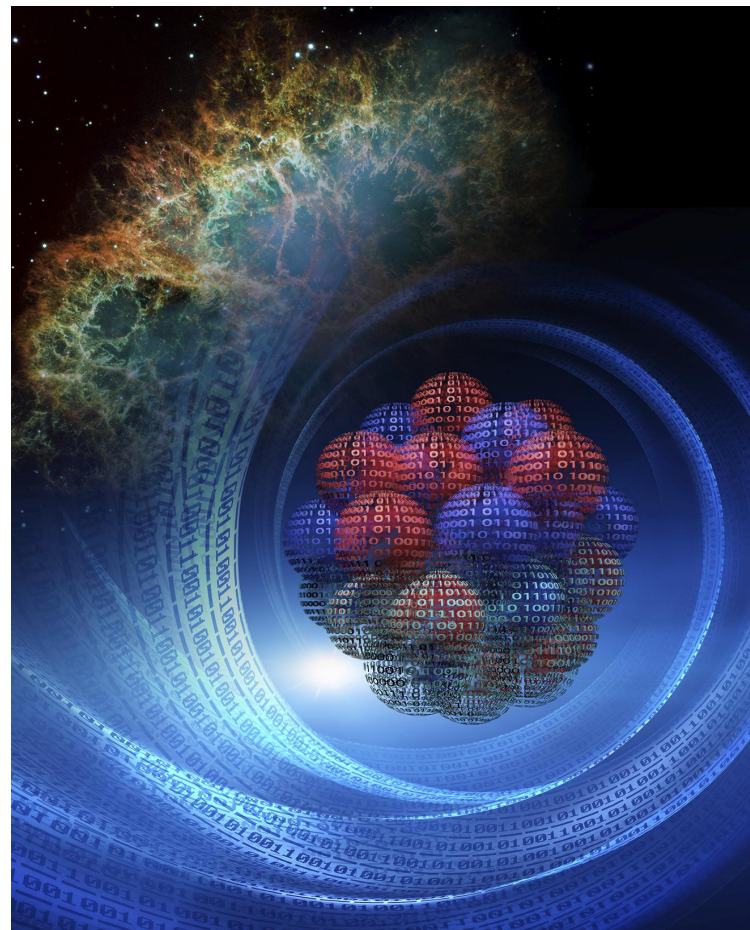
Large-scale = predictions for **thousands** of nuclei and many **observables**.

**BSkG1** and **BSkG2** are the latest models

- full three-dimensional representation of nuclei
- includes **triaxial deformation!**
- and **time-reversal** symmetry breaking => new observables!
- data available to all!

**BSkG2:** W. Ryssens et al. EPJA 58, 246 (2022).

**BSkG1:** G. Scamps et al., EPJA 57, 333 (2021).



## Available:

1. **BSkG2, fission:** W. Rysseus et al., arXiv:2302.03097 [nucl-th].
2. **BSkG2, g.s. properties:** W. Rysseus et al. EPJA 58, 246 (2022).
3. **BSkG1, g.s. properties:** G. Scamps et al., EPJA 57, 333 (2021).

Coming soon(-ish) on <http://www.astrou.ulb.ac.be/bruslib/>

Regular Article - Theoretical Physics | Published: 16 December 2021

## Skyme-Hartree-Fock-Bogoliubov mass models on a 3D mesh: effect of triaxial shape

Guillaume Scamps , Stéphanie Goriely, Erik Olsen, Michael Bender & Wouter Rysseus

The European Physical Journal A 57, Article number: 333 (2021) | [Cite this article](#)

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This article has been updated

### Abstract

The modelling of nuclear reactions and radioactive decays in astrophysical or earth-based conditions requires detailed knowledge of the masses of essentially all nuclei. Microscopic mass models based on nuclear energy density functions (EDFs) can be descriptive and used to provide this information. The concept of intrinsic viscosity breaking is central to the predictive power of EDF approaches, yet is generally not exploited to the utmost by mass models because of the computational demands of adjusting up to about two dozen parameters to thousands of nuclear masses. We report on a first step to bridge the gap between what is presently feasible for studies of individual nuclei and large-scale models: we present a new Skyme-EDF-based model that was adjusted using a three-dimensional coordinate-space representation, for the first time allowing for both axial and triaxial deformations during the adjustment process. To compensate for the substantial increase in computational cost brought by the latter, we have employed a committee of multi-layer neural networks to model the objective function in parameter space and guide us towards the overall best fit. The resulting mass model BSkG1 is computed with the EDF model independently of the neural network. It yields a root mean square (rms) deviation on the 2457 known masses of 741 keV and an rms deviation on the 884 measured charge radii of 0.024 fm.

### 1 Introduction

Regular Article - Theoretical Physics | Published: 16 December 2021

## Skyme-Hartree-Fock-Bogoliubov mass models on a 3D mesh: effect of triaxial shape

Guillaume Scamps , Stéphanie Goriely, Erik Olsen, Michael Bender & Wouter Rysseus

The European Physical Journal A 57, Article number: 333 (2021) | [Cite this article](#)

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Sections: Abstract, Introduction, Ingredients of the mass model, Model adjustment using neural networks, The BSkG1 parameterization, Conclusions and outlook, Data Availability Statement, Change history, Notes, References, Acknowledgements, Author information, Additional information, Supplementary Information, Appendices, Rights and permissions, About this article

Z	N	Mag[MeV]	BSkG1[MeV]	BSkG2[MeV]	BSkG3[MeV]	beta0	beta2	beta4	beta6	beta8	beta10	beta12	beta14	beta16	beta18	beta20	beta22	beta24	beta26	beta28	beta30	beta32	beta34	beta36	beta38	beta40	beta42	beta44	beta46	beta48	beta50	beta52	beta54	beta56	beta58	beta60	beta62	beta64	beta66	beta68	beta70	beta72	beta74	beta76	beta78	beta80	beta82	beta84	beta86	beta88	beta90	beta92	beta94	beta96	beta98	beta100	beta102	beta104	beta106	beta108	beta110	beta112	beta114	beta116	beta118	beta120	beta122	beta124	beta126	beta128	beta130	beta132	beta134	beta136	beta138	beta140	beta142	beta144	beta146	beta148	beta150	beta152	beta154	beta156	beta158	beta160	beta162	beta164	beta166	beta168	beta170	beta172	beta174	beta176	beta178	beta180	beta182	beta184	beta186	beta188	beta190	beta192	beta194	beta196	beta198	beta200	beta202	beta204	beta206	beta208	beta210	beta212	beta214	beta216	beta218	beta220	beta222	beta224	beta226	beta228	beta230	beta232	beta234	beta236	beta238	beta240	beta242	beta244	beta246	beta248	beta250	beta252	beta254	beta256	beta258	beta260	beta262	beta264	beta266	beta268	beta270	beta272	beta274	beta276	beta278	beta280	beta282	beta284	beta286	beta288	beta290	beta292	beta294	beta296	beta298	beta300	beta302	beta304	beta306	beta308	beta310	beta312	beta314	beta316	beta318	beta320	beta322	beta324	beta326	beta328	beta330	beta332	beta334	beta336	beta338	beta340	beta342	beta344	beta346	beta348	beta350	beta352	beta354	beta356	beta358	beta360	beta362	beta364	beta366	beta368	beta370	beta372	beta374	beta376	beta378	beta380	beta382	beta384	beta386	beta388	beta390	beta392	beta394	beta396	beta398	beta400	beta402	beta404	beta406	beta408	beta410	beta412	beta414	beta416	beta418	beta420	beta422	beta424	beta426	beta428	beta430	beta432	beta434	beta436	beta438	beta440	beta442	beta444	beta446	beta448	beta450	beta452	beta454	beta456	beta458	beta460	beta462	beta464	beta466	beta468	beta470	beta472	beta474	beta476	beta478	beta480	beta482	beta484	beta486	beta488	beta490	beta492	beta494	beta496	beta498	beta500	beta502	beta504	beta506	beta508	beta510	beta512	beta514	beta516	beta518	beta520	beta522	beta524	beta526	beta528	beta530	beta532	beta534	beta536	beta538	beta540	beta542	beta544	beta546	beta548	beta550	beta552	beta554	beta556	beta558	beta560	beta562	beta564	beta566	beta568	beta570	beta572	beta574	beta576	beta578	beta580	beta582	beta584	beta586	beta588	beta590	beta592	beta594	beta596	beta598	beta600	beta602	beta604	beta606	beta608	beta610	beta612	beta614	beta616	beta618	beta620	beta622	beta624	beta626	beta628	beta630	beta632	beta634	beta636	beta638	beta640	beta642	beta644	beta646	beta648	beta650	beta652	beta654	beta656	beta658	beta660	beta662	beta664	beta666	beta668	beta670	beta672	beta674	beta676	beta678	beta680	beta682	beta684	beta686	beta688	beta690	beta692	beta694	beta696	beta698	beta700	beta702	beta704	beta706	beta708	beta710	beta712	beta714	beta716	beta718	beta720	beta722	beta724	beta726	beta728	beta730	beta732	beta734	beta736	beta738	beta740	beta742	beta744	beta746	beta748	beta750	beta752	beta754	beta756	beta758	beta760	beta762	beta764	beta766	beta768	beta770	beta772	beta774	beta776	beta778	beta780	beta782	beta784	beta786	beta788	beta790	beta792	beta794	beta796	beta798	beta800	beta802	beta804	beta806	beta808	beta810	beta812	beta814	beta816	beta818	beta820	beta822	beta824	beta826	beta828	beta830	beta832	beta834	beta836	beta838	beta840	beta842	beta844	beta846	beta848	beta850	beta852	beta854	beta856	beta858	beta860	beta862	beta864	beta866	beta868	beta870	beta872	beta874	beta876	beta878	beta880	beta882	beta884	beta886	beta888	beta890	beta892	beta894	beta896	beta898	beta900	beta902	beta904	beta906	beta908	beta910	beta912	beta914	beta916	beta918	beta920	beta922	beta924	beta926	beta928	beta930	beta932	beta934	beta936	beta938	beta940	beta942	beta944	beta946	beta948	beta950	beta952	beta954	beta956	beta958	beta960	beta962	beta964	beta966	beta968	beta970	beta972	beta974	beta976	beta978	beta980	beta982	beta984	beta986	beta988	beta990	beta992	beta994	beta996	beta998	beta1000
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# Conclusion

We build large-scale, microscopic models of nuclear structure for applications.

Microscopic = with **simple** wave functions yet complex **symmetry breaking**.

Large-scale = predictions for **thousands** of nuclei and many **observables**.

**BSkG1** and **BSkG2** are the latest state-of-the-art models entries

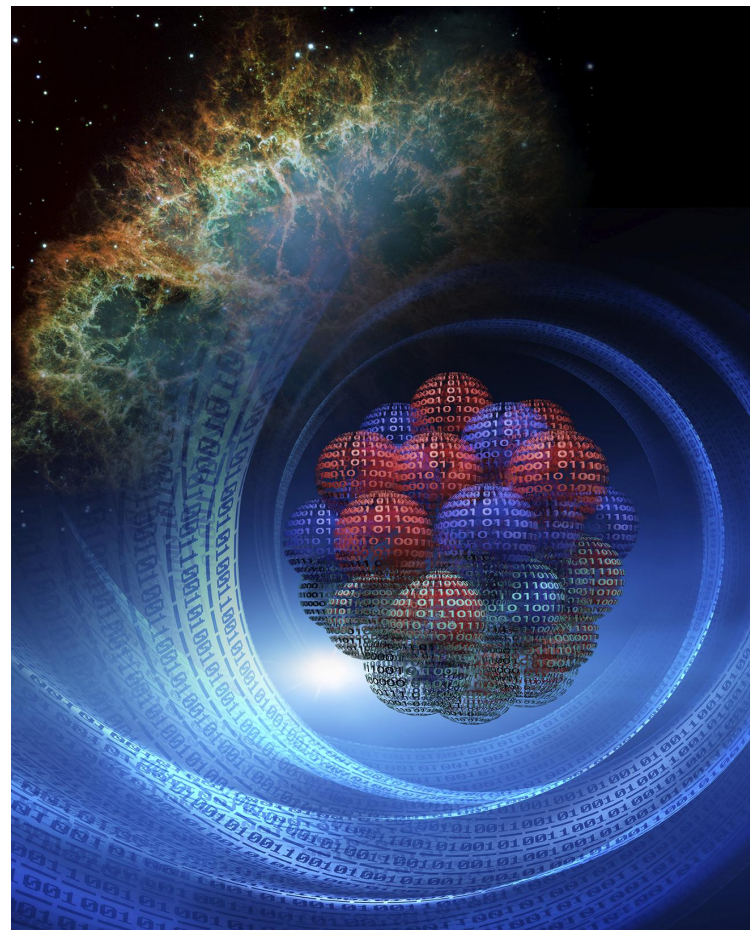
- full three-dimensional representation of nuclei
- includes **triaxial deformation** !
- **time-reversal** symmetry breaking => new observables!
- data available to all!

**BSkG2:** W. Ryssens et al. EPJA 58, 246 (2022).

**BSkG1:** G. Scamps et al., EPJA 57, 333 (2021).

In our project, I hope you want to help with:

1. providing data for our parameter fits
2. telling us where wave functions need refining
3. helping us refine the form of the EDF



# Problems: what we cannot do (yet)!

1. “**Single-particle**” properties are deficient
  - a. g.s. spins
  - b. details of charge radii
  - c. magnetic moments
2. The **EDF form** is phenomenological:
  - a. no **systematic** improvement possible
  - b. no **connection** to the **nucleon-nucleon** interaction
  - c. no way to provide **systematic** errors
3. We have no predictions for thousands of nuclei:
  - a.  $\beta$ -decay rates
  - b. level-schemes + transition rates
  - c. magnetic moments

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3. We have no systematic predictions for
  - a.  $\beta$ -decay
  - b. level-schemes / transition rates / half-lives
  - c. magnetic moments

We need to work on:

1. extendable, non-empirical EDF forms.
2. beyond-mean-field at the scale of the nuclear chart.

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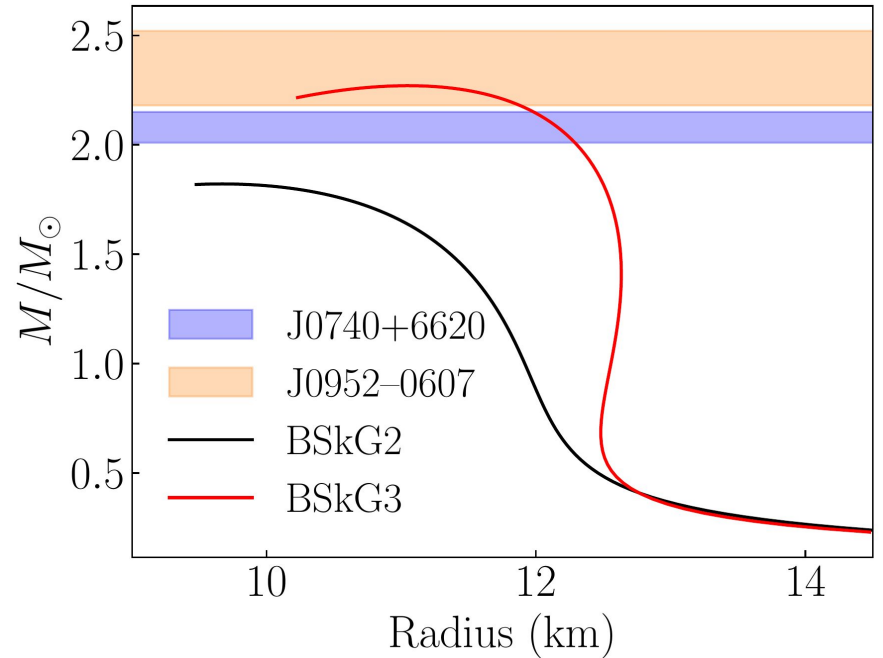
# Outlook

## BSkG3

- compatible with the existence of heavy pulsars
- without deteriorating structure properties of BSkG1/2

## Databases based on BSkG2:

- nuclear level densities
- fission properties
- details of (charge) densities



E. Fonseca et al., ApJL 915, L12 (2021)

R. W. Roman et al., ApJL 934, L17 (2022).



# Thank you for...

..... all the wonderful work!



S. Goriely  
G. Grams  
N. Chamel  
N. Shchepochin



M. Bender  
J. Bonnard



G. Scamps



M. Hukkanen  
M. Stryjczyk  
A. Kankainen



P. Ascher  
S. Grévy



S. Sels  
E. Verstraelen  
T. Cocolios  
P. Van Duppen



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G. Giacalone



B. Schenke  
C. Shen

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..... the computing time!

The logo for CÉCI features the text 'CÉCI' in white, bold, sans-serif font on a blue rounded rectangular background.



# Thank you for...

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N. Shchечilin



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C. Shen

..... the computing time!



..... the funding!



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N. Shchечilin



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J. Bonnard



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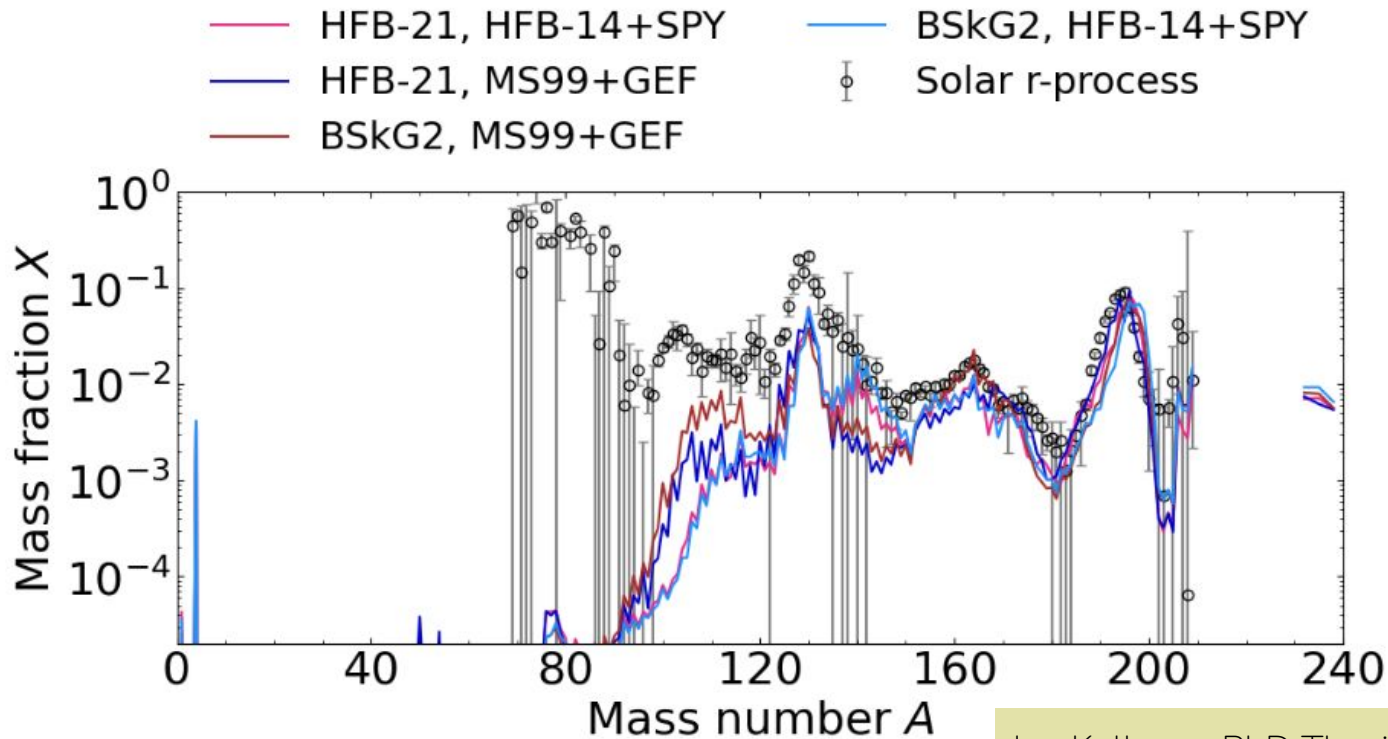


..... the funding!



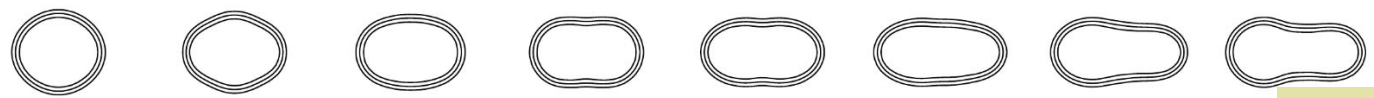
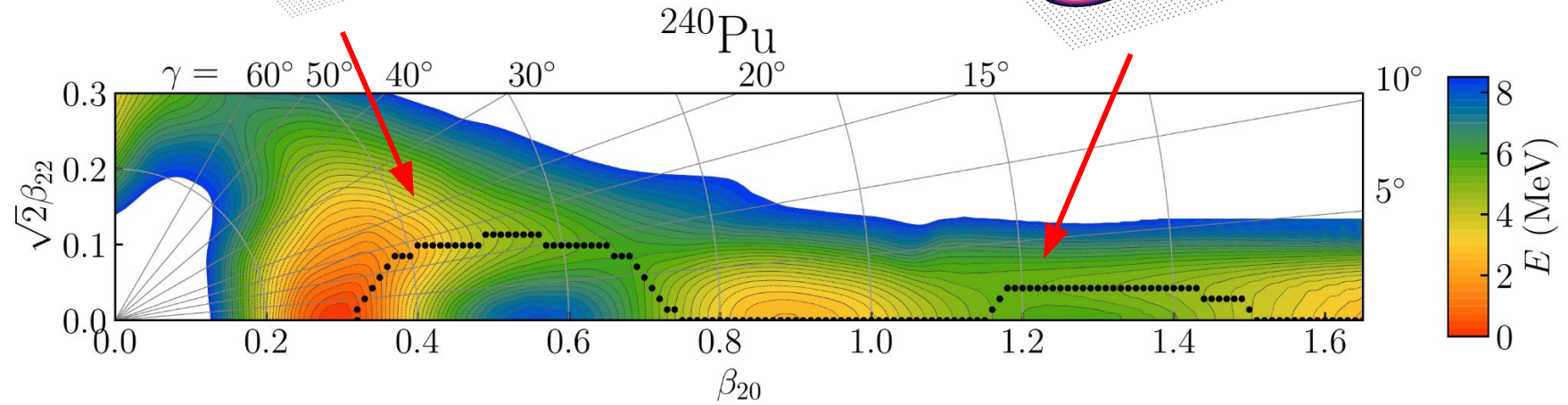
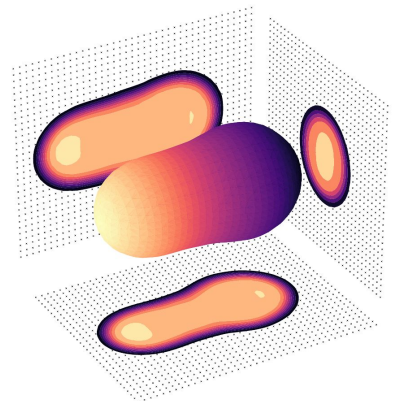
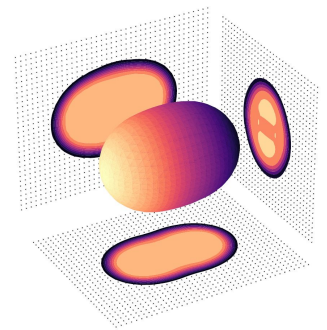
..... your attention!

Bonus!



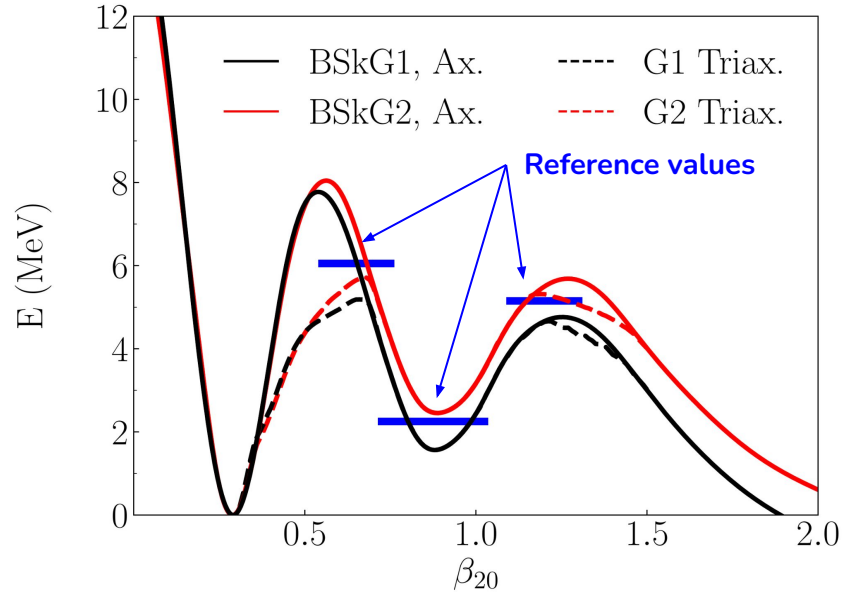
Ina Kullman, PhD Thesis (2022)

# Observables: fission



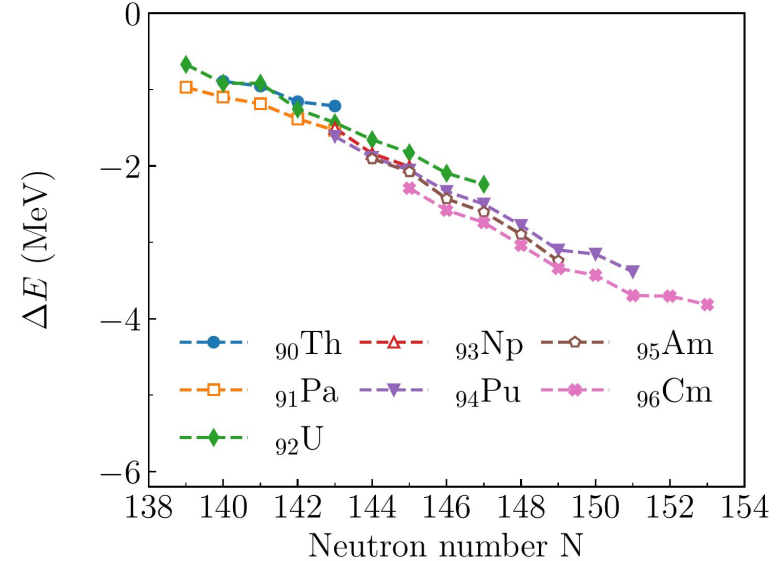
# Observables: fission

R. Capote et al., Nuclear Data Sheets **110**, 3107 (2009).



## Triaxial deformation for $^{240}\text{Pu}$

- Large effect on inner barrier
- No effect on isomers
- Modest effect on outer barrier



## Triaxial deformation for actinides

- Larger effects with growing N
- reminder:  $\sigma(\text{fission}) < 0.5 \text{ MeV}$
- what other regions does it affect?



